Ah, life could be a dream . . . .
Product Rule, Quotient Rule, Chain Rule
Exponential Functions
And, if that weren't enough
(and don't you think it ought to be)
Inverse Trig Functions !
. . . . Sha Boom, Sha Boom

ADVANCED PLACEMENT CALCULUS AB CHAPTER 5

2002

Yes. You do need those old trig notes.
You do still
love Trig,
don't you?

EXAM Chapters 1 - 4
 # 1 R/C Pg 103 - 108
 # 2 Pg 109 1 - 6, 10
 # 3 Pg 114 - 115
1 - 11, 17 - 19, 26
On pblm 4;*No Decimals*
Ln 2 = A; Ln 3 = B **only!**
Find decimal ans. for prob 11
Der of sine & Cos
10/14

#4 Pg 118 1, 4 - 7, 11 - 14

5 R/C Pg 143 - 146.

Der of Log
R/C is about inv fctns
Lecture in notebook on lined paper
10/15

6 & # 7
Pg 125 - 126 16 - 27, 30, 31
Pg 131 - 132 23, 24, 26, 28
Pg 142 1 - 31 odd
for 25-31 simplify completely
#8 "I Know TaShun"

Do Not Use $(\ln x)(\ln x)$ for $(\ln x)^2$ **use:** $\ln^2 x$

Product & quotient rules & Chain Rule
10/16

10/17

9 Pg 142 2 - 32 even
for pblms 26-32: **simplify completely.**

put pg 125 16 - 27 and pg 142 1 - 31 odd on boards.

10/1

 # 10 Pg 146 1 - 4, 8 - 10, 13 - 21 odd; 24, 26
also sketch each of the 6 **inverse** trig fctns and state **domain** & range.
also: simplify and/or solve:

$\sin^{-1}(\sin u) = u$

$\tan(\sin^{-1} u) = ?$

10/ 21

I show the 6 inv trg fctns. They must take good notes

Continue on "CE, Third Time"

On boards:
52 – 55, 61, 62, 68, 63,
24 - 27, 29, 30, 33

10/ 28

11 Pg 152 - 153
1 - 21 odd, 22, 28
Der of exponential fctns
 # 12, 13 Explain the formula at the bottom of pg 147. Use the 2 examples supplied **below** to show your understanding of the concept..

Very important!

See HW #12, #13

$f(x) = \sqrt{x}$ for example # 1

$f(x) = e^x$ for example # 2

10/29

14 Pg 159 2 - 28
15 Pg 162 GQ CM 2 - 5
Pg 162
GQ Con 1, 3, 6, 8 –11
Pg 163 RE 1 - 7
#16 Compare & contrast thm3 pg 157-158 with lecture.
*See note in lower rt corner of sked. 10/23

Deriv of inverse trig fctns

10/30

17 Pg 163 RE 8 - 23, 50

Put on boards:
Pg 159 2 - 28
Pg 162 2 – 5
Pg 162 GQ Con 3, 8, 9, 10
Pg 163 2 – 7

EXAM CHAPTER 5

10/31

18,
Pg 163 24 – 27, 29, 30, 33
Pg 164 52 - 58, 61, 62, 63, 66, 68
pblm 52 evaluate at x=5 no

$x_0 = 5$ 10/

 # 19, 20, 21, 22, 23
Pick up "3rd Time"
Put on boards all of yesterday's HW.
Emphasize problem 50.

On day when #14,15 assigned,
Require HW #12,13 turned in before scho to grade and discuss i class that day. No lat considerations

5.1

Pg 103-105

(1) $D(5x^3+6x-2)\Big]_{x=-1} = 15x^2+6\Big]_{x=-1} = 9$

(2) ↓ Routine

(16) $y = f(v) = 2v^2+3v$

$\frac{dy}{dv}$ using $\lim_{v_1 \to v} \frac{f(v_1)-f(v)}{v_1-v}$

$$\lim_{v_1 \to v} \frac{2v_1^2+3v_1-2v^2-3v}{v_1-v} =$$

$$\lim_{v_1 \to v} \frac{2(v_1^2-v)+3(v_1-v)}{v_1-v} =$$

$$\lim_{v_1 \to v} 2(v_1+v)+3 = 4v+3$$

↓

(18) $y = -16t^2+80t$

(19) a) $\frac{dy}{dt} = -32t+80$

b) $-16(2t-5)$ $\quad y'>0 \ \forall\ t < \frac{5}{2}$

$y'<0 \ \forall\ t > \frac{5}{2}$

$y'=0 \quad t=\frac{5}{2}$

c) Velocity at $t=0$ is 80 meters/sec.

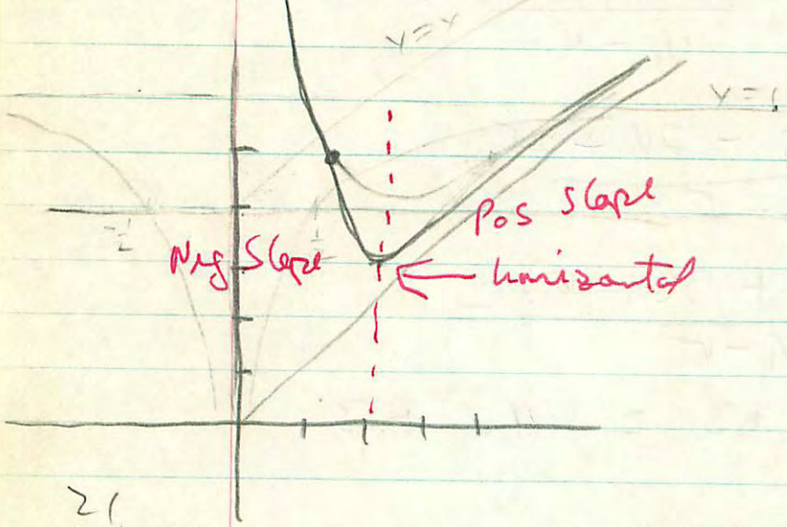
(20) $f(x) = x + \frac{4}{x}$ $\lim_{x_1 \to x} \frac{f(x_1) - f(x)}{x_1 - x}$

$$\lim_{x_1 \to x} \frac{x_1 + \frac{4}{x_1} - x - \frac{4}{x}}{x_1 - x} = \lim_{x_1 \to x} \frac{\frac{x_1^2 + 4}{x_1} - \frac{x^2 + 4}{x}}{x_1 - x}$$

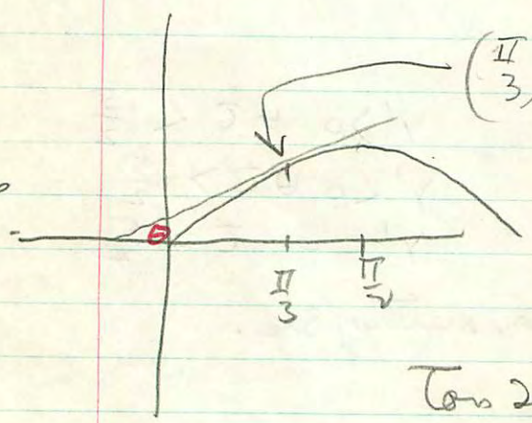
$$\lim_{x_1 \to x} \frac{x_1^2 x + 4x - x^2 x_1 - 4x_1}{x_1 x (x_1 - x)} = \lim_{x_1 \to x} \frac{x_1 x (x_1 - x) - 4(x_1 - x)}{x_1 x (x_1 - x)}$$

$$\lim_{x_1 \to x} \frac{x_1 x - 4}{x_1 x} = \frac{x^2 - 4}{x^2} = 1 - \frac{4}{x^2}$$

$*^2$



21.



#6

$Y = \sin x$

$Y' = \cos x \Big|_{\frac{\pi}{6}} = \frac{1}{2} = \tan\theta$

$\tan 26^\circ = .4877$

$27^\circ = .5095$

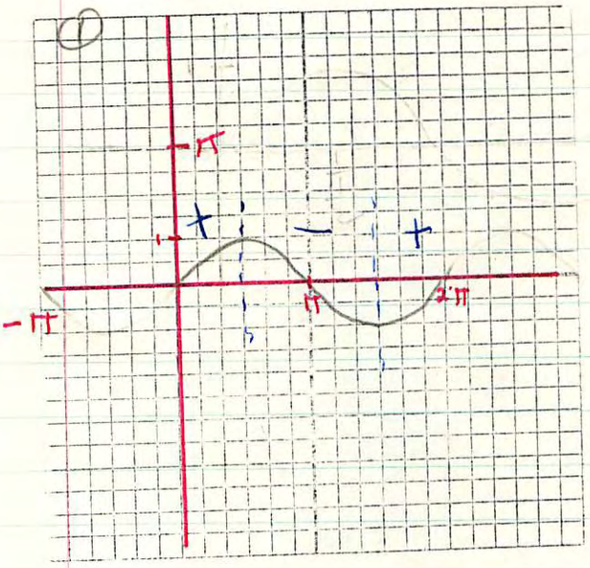
$\therefore \theta \approx 26.5$

5.2

p. 105 - 109

We have Proven $d(\sin x) = \cos x$ etc.

in Hw rg 90 phlms 19, 20



② a) $\frac{\sqrt{2}}{2}$, b) $\frac{\sqrt{3}}{2}$, 0, -1 c) D)

③ routine

④ a) $\cos x\big]_{x=0} = 1$

b) $\cos x\big]_{x=\frac{\pi}{4}} = \frac{\sqrt{2}}{2}$

c) $\cos x\big]_{x=\frac{\pi}{2}} = 0$

⑤

	$t=0$	$t=\frac{\pi}{2}$	$t=\pi$	$t=\frac{3\pi}{2}$
a)	0, Even	1, 1 unit above	0	-1, 1 unit below
b)	Vel up 1 unit/sec	Stopped	Vel -1 down 1 unit/sec	Stopped

c) Note speed does not have direction ∴ |velocity| = speed.

⑥ a) $\cos\frac{\pi}{3} = \frac{1}{2}$ b) $\tan\theta = \pm\frac{1}{2} \Rightarrow 26.57°$

⑦ Proof of $d(\cos x) = -\sin x$

⑩ $\lim_{x_1 \to x} \frac{\sin 3x_1 - \sin 3x}{x_1 - x} = \lim_{x_1 \to x} \frac{\sin(2x_1 + x_1) - \sin(2x + x)}{x_1 - x} =$

Try again.

$\lim_{x_1 \to x} \frac{\sin 2x_1 \cos x_1 + \cos 2x_1 \sin x_1 - \sin 2x \cos x - \sin x \cos 2x}{x_1 - x}$

$\lim_{x_1 \to x} \frac{2\sin x_1 \cos^2 x_1 + \cos^2 x_1 \sin x_1 - \sin^3 x_1 - 2\sin x \cos^2 x + \sin^3 x - \cos^2 x \sin x}{x_1 - x}$

$\lim_{x_1 \to x} \frac{3\sin x_1 \cos^2 x_1 - \sin^3 x_1 + \sin^3 x - 3\sin x \cos^2 x}{x_1 - x}$

(10) $\lim_{h\to 0} \frac{\sin 3(x+h) - \sin 3x}{h} =$ agem aurT Pg

$\lim_{h\to 0} \frac{\sin(2(x+h)+(x+h)) - \sin 3x}{h} =$

$\lim_{h\to 0} \frac{\sin 2(x+h)\cos x+h + \cos 2(x+h)\sin(x+h) - \sin 3x}{h}$

$\lim_{h\to 0} \frac{2\sin(x+h)\cos^2(x+h) + \cos^2(x+h)\sin(x+h) - \sin^3(x+h) - \sin 3x}{h}$

$\lim_{h\to 0} \frac{3\sin(x+h)\cos^2(x+h) - \sin(x+h)(\sin^2(x+h)) - \sin 3x}{h}$

$\lim_{h\to 0}$ $3\sin x \cos h \cos^2(x+h) + 3\sin h \cos x \cos^2(x+h) +$

$-\sin x \cos h \sin^2(x+h) - \cos x \sin h \sin^2(x+h) - \sin 3x$

$\lim_{h\to 0}$ $\sin x(\cos h \cos^2(x+h) - \cos h$

$\sin 3x = \sin 2x + x =$ ~~$2\sin x \cos$~~

$\sin 2x \cos x + \cos 2x \sin x$

$2\sin x \cos^2 x + \cos^2 x \sin x - \sin^3 x$

$\lim_{h\to 0} \frac{\sin(3x+3h) - \sin 3x}{h} \Rightarrow$

$\frac{3(\quad)}{3h}$

10

$$\lim_{h\to 0} \frac{\sin(3x+3h) - \sin 3x}{h} = \lim_{h\to 0} \frac{\sin 3x\cos 3h + \cos 3x \sin 3h - \sin 3x}{h}$$

$$\lim_{h\to 0} \frac{\sin 3x(\cos 3h - 1) + \cos 3x \sin 3h}{h}$$

$$\lim_{h\to 0} -3\sin 3x \frac{(1-\cos 3h)}{3h} + 3\cos 3x \frac{\sin 3h}{3h} = 3\cos 3x.$$

(13) $y = 2\sin t$

a) $y' = 2\cos t$

b) low tide is at $t = \frac{3\pi}{2}$ hrs, which is -2.

$y']_{\frac{3\pi}{2}} = 0$ at mean tide

when $t = 0$ $y']_0 = 2$

(16) $D\left(\frac{1-\cos 2x}{2}\right) = \sin^2 2x$

$$D\left(\frac{1-\cos^2 x + \sin^2 x}{2}\right) = D\left(\frac{2\sin^2 x}{2}\right) = D(\sin^2 x)$$

$$\lim_{h\to 0} \frac{1-\cos(2x+2h)}{2h} - \frac{1-\cos 2x}{2h} = \lim_{h\to 0} \frac{-\cos(2x+2h)+\cos 2x}{2h}$$

$$\lim_{h\to 0} \frac{-\cos 2x \sin 2h - \cos 2h \sin 2x + \cos 2x}{2h} =$$

$$\lim_{h\to 0} \frac{-\cos 2x(\sin 2h - 1) + \cos 2h \sin 2x}{2h}$$

Part a.

(26) $D = K \log\left(\frac{\frac{1}{2}P}{P_0}\right) = K \log\left(\frac{P}{2P_0}\right) = K(\log P - \log P_0 - \log 2) =$

$K(\log P - \log P_0) - K \log 2 =$

$K\left(\log \frac{P}{P_0}\right) - K \log 2$

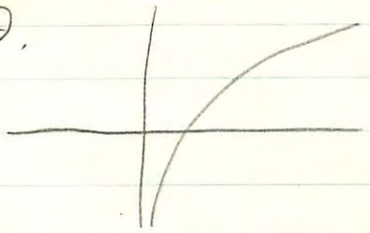
↑

Constant & decrease.

5.3
Pg 109-115

① a) $\log_2 32 = 5$ b) etc.

②.



$-\infty$, $+\infty$

③ these are reciprocals $\log_b a = \frac{1}{\log_a b}$

~~④~~

5 routine.

↓

9

10 $\log_4 A = 2.1$

a) $\log_4 A^2 = 4.2$ b) $\log_4 64A = \log_4 64 + \log_4 A$

$4 + 2.1 = 6.1$

c) $\log_{04} \frac{16}{A} = \log_4 16 - \log_4 A = 2 - 2.1 = -.1$

d) $\log_2 A = \frac{\log_4 A}{\log_4 2} = \frac{2.1}{\frac{1}{2}} = 4.2$

⑰ Routine.

↓

20

~~21~~ $a, b > 0$ $\log_b a \cdot \log_a b = 1$

26. Given $D = K \log \frac{P}{P_0}$

$K \log \frac{\frac{1}{2}P}{P_0} = K \log \frac{P}{2P_0} = K(\log P - \log 2P_0)$

$K \log P - K \log 2P_0$

↑ all constants this is the fixed amount

over. ↓

Solve for P in terms of D.

$$D = K \log \frac{P}{P_0}$$

$$\frac{D}{K} = \log \frac{P}{P_0}$$

$$\frac{P}{P_0} = a^{\frac{D}{K}}$$

$$P = P_0 a^{\frac{D}{K}}$$ a is base of log.

$$\lim_{h\to 0} \frac{\log(x+h) - \log x}{h}$$

Log Properties reviewed previous day (when exponents were returned)

$$\lim_{h\to 0} \frac{1}{h}\left(\log(x+h) - \log x\right)$$

$$\lim_{h\to 0} \frac{1}{h}\left(\log\left(\frac{x+h}{x}\right)\right)$$ 1ST do Log Properties

$$\lim_{h\to 0} \frac{1}{h}\log\left(1+\frac{h}{x}\right)$$ $\log(ab) = \log a + \log b$

$$\lim_{h\to 0} \log\left(1+\frac{h}{x}\right)^{\frac{1}{h}}$$ $\log \frac{a}{b} = \log a - \log b$

$$\lim_{h\to 0} \log\left[\left(1+\frac{h}{x}\right)^{\frac{x}{h}}\right]^{\frac{1}{x}}$$ $\log a^b = b \log a$

$$\lim_{h\to 0} \frac{1}{x}\log\left(1+\frac{h}{x}\right)^{\frac{x}{h}}$$ $\log_b a = x$ means $b^x = a$

$\frac{1}{x}\log e$ or $\frac{1}{x}\log_{10} e$

Logs and exponents and how exponent properties

If base of log had been e

$$\frac{1}{x}\log_e e$$

$\frac{1}{x}$ which is the reason we use base e.

DNE

How do we know

$\ln 2 > \frac{1}{2}$?

start w/ something obvious then arrive at WTP

we know $e < 4$

$\ln e < \ln 4$

$1 < \ln 2^2$

$1 < 2 \ln 2$

$\frac{1}{2} < \ln 2$

5.4
Pg 115–119

Pg 118

(1) a) $\frac{1}{x}\log_{10} e$ b) $\frac{1}{x}\log_2 e$

c) $\frac{1}{x}\log_5 e$ d) $\frac{1}{x}$ e) $\frac{1}{x}$

(4) a) $\ln\frac{1}{2} = -\ln 2 \approx -.69$

b) $\ln 6 = \ln(2\cdot 3) = \ln 2 + \ln 3 = .69 + 1.10 = 1.79$

c) $\ln 12 = 2\ln 2 + \ln 3 = 1.38 + 1.10 = 2.48$

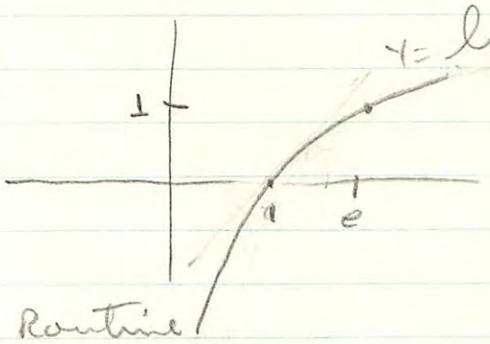
d) $\ln\frac{2}{3} = \ln 2 - \ln 3 = .69 - 1.10 = -.41$

e) $\ln 1.5 = \ln\frac{3}{2} = 1.10 - .69 = .41$

f) $\ln 2^7 = 7\ln 2 = (.69)7 = 4.83$

g) $\ln\frac{1}{9} = \ln 3^{-2} = -2\ln 3 = -2.20$

5)



Slope of tangent at $x=1$

$\frac{1}{x}\Big]_{x=1} = 1$

$\tan\theta = 1 \Rightarrow 45^\circ = \theta$.

6. Routine

7. a) $\frac{1}{x}\Big]_{x=2} = \frac{1}{2}$ ∴ image has length $\frac{1}{2}$

b) $\ln 2.08 - \ln 2 = .03922$

11. a) $\frac{1}{x}\Big]_{x=3} = \frac{1}{3}$ b) $\ln 3.06 - \ln 3 = 1.12 - 1.099 = .02$

12. ~~$\ln x = \frac{1}{x} \le \ln 0$~~

wts $f(x) = \ln x - \frac{1}{x}$ wts $f(\bar{x}) = 0$

$f(1) = -1$

$f(2) = \ln 2 - \frac{1}{2} = .193$

implies $\ln x - \frac{1}{x} = 0$ which means $\ln x = \frac{1}{x}$ for that $\bar{x}$.

$f(a) < 0 < f(b)$ by intermediate value thm.

f is continuous over $[1,2]$ ∴ $f(1) < 0$ & $f(2) > 0$ so there is a $\# \bar{x} \ni f(\bar{x}) = 0$ but if $f(\bar{x}) = 0$ This

(13) $d(\ln x) = \frac{1}{x}$ $\quad d(\sin x) = \cos x$

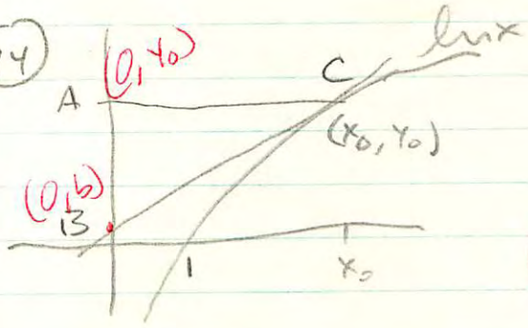
let $f(x) = \cos x - \frac{1}{x}$ $\quad$ WTS $f(X) = 0$

let $a = \pi \quad b = 2\pi$

$f(a) = -1 - \frac{1}{\pi} < 0 \qquad f(a) < 0 < f(b)$

$f(b) = 1 - \frac{1}{2\pi} > 0$ $\quad$ So by intermediate Value thm $\exists X \ni f(X) = 0$

(14)



WTP $\overline{AB} = 1$

$AC \Rightarrow y = y_0$

$BC \Rightarrow y = \frac{1}{x_0}x + B$ $\quad$ Not Necessary.

$(0, B) \quad (x_0, y_0)$

$\frac{y_0 - B}{x_0} \overset{\text{set}}{=} \frac{1}{x_0} \Rightarrow y_0 - B = 1 \Rightarrow \overline{AB} = 1$

(15) $\lim_{x_1 \to x} \frac{x_1 \ln x_1 - x \ln x}{x_1 - x} = \lim_{x_1 \to x} \frac{x_1 \ln x_1 - x_1 \ln x + x_1 \ln x - x \ln x}{x_1 - x}$

$x_1 - x \;\big|\; \frac{x_1}{x_1} - \frac{x}{x_1}$

$1 + \frac{x}{x_1} - x$

$\lim_{x_1 \to x} \frac{x_1(\ln x_1 - \ln x) + \ln x(x_1 - x)}{x_1 - x} =$

$\lim_{x_1 \to x} \ln x + \frac{x_1(\ln x_1 - \ln x)}{x_1 - x} = \lim_{x_1 \to x} \frac{x_1 \left(\ln \frac{x_1}{x}\right)}{x_1 - x} + \ln x$

$\lim_{x_1 \to x} \frac{x_1}{x_1 - x} \ln \frac{x_1}{x} + \ln x$

Derivatives of Sum, Diff, Prod & Products.

$$d(u+v) = u' + v'$$

$$d(u-v) = u' - v'$$

$$d(uv) = v\,du + u\,dv$$

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$d(f(g)) = f'(g(x))\,g'(x)$$ See next section

Der of "what's there" (TIMES) Der of "inside"

nice to know (maybe to be developed)

$$(\tan x)' = \sec^2 x$$

$$(\sec x)' = \sec x \tan x$$

$$(\cot x)' = -\csc^2 x$$

$$(\csc x)' = -\csc x \cot x$$

$$(\ln x)' = \frac{1}{x}$$

$$(e^x)' = e^x$$

5.5

119 - 126

① $\frac{d}{dx}(\cos x + \sin x) = -\sin x + \cos x$

② $\frac{d}{dx}(2x^3 - 3\cos x) = 6x^2 + 3\sin x$

↓ Routine.

27

30 a) $x^2 \sin x \ln x$ using formula in Rule 29.

$2x \sin x \ln x + x^2 \cos x \ln x + x \sin x$.

b) $x^3 \cos x (1 + \sin x)$

$3x^2 \cos x (1 + \sin x) - x^3 \sin x (1 + \sin x) + x^3 \cos^2 x$

31 Routine.

5.6

P. 126-132

Derivative of Quotient

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

Nice to Know

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = -\csc x \cot x$

I Know TaShun!

Given: $y = f(x)$; $f(3) = 5$; $f'(3) = 7$; $f''(3) = \frac{1}{2}$

Evaluate each of the following at $x = 3$:

a) $\frac{dy}{dx} = f'(3) = 7$ b) $\frac{dy^2}{dx}$ $(y^2)' = 2yy'$ $2 \cdot 5 \cdot 7 = 70$ c) $\frac{d^2y}{dx^2} = \frac{1}{2}$

d) $\frac{dy}{dx^2} = \frac{7}{6}$ e) $\frac{dy^2}{dx^2}$ f) $\frac{dy}{dx}$ g) $\frac{dy^2}{dx}$

h) $\frac{dy^2}{dx^2}$ i) $\frac{dy}{dx^2}$

$\frac{dy}{dx} \cdot \frac{dx}{dx^2} = \frac{dy}{dx} \div \frac{dx^2}{dx}$

$\frac{dy^2}{dx^2}$

$\frac{\frac{dy}{dx}}{\frac{dx^2}{dx}} \rightarrow \frac{f'(3)}{2x} \rightarrow \frac{7}{2 \cdot 3} = \frac{7}{6}$

$\frac{\frac{dy^2}{dx}}{\frac{dx^2}{dx}}$ from part B $\frac{70}{2 \cdot 3} = \frac{70}{6} = \frac{35}{3}$

5.6

P. 131

① $d\left(\frac{2x+1}{3x+2}\right) = \frac{2(3x+2) - 3(2x+1)}{(3x+2)^2} = \frac{1}{(3x+2)^2}$

↓ 22 Routine,

(23) $\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$

$\frac{d(f^{-1})}{dx} = -f^{-2}\frac{df}{dx}$

$\frac{-f'}{f^2}$

(24) $(\cot x)' = -\csc^2 x$

$\left(\frac{1}{\tan x}\right)' = \frac{\sec^2 x}{\tan^2 x} = \sec^2 \cot^2 x =$

$\frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \csc^2 x$

$(\csc x)' = -\csc x \cot x$

$\left(\frac{1}{\sin x}\right)' = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x} =$

$-\csc x \cot x.$

(26) $f(x) = x + \frac{3}{x} \qquad x > 0$

a)
$f(.1) = .1 + 30 = 30.1$
$f(1) = 1 + 3 = 4$
$f(10) = 10 + .3 = 10.3$

b) $\lim_{x \to \infty} x + \frac{3}{x} = +\infty$ $\lim_{x \to 0^+} x + \frac{3}{x} \Rightarrow +\infty$

c) $f'(x) = 1 - \frac{3}{x^2}$

d) $x^2 - 3 \overset{\text{Set}}{=} 0$

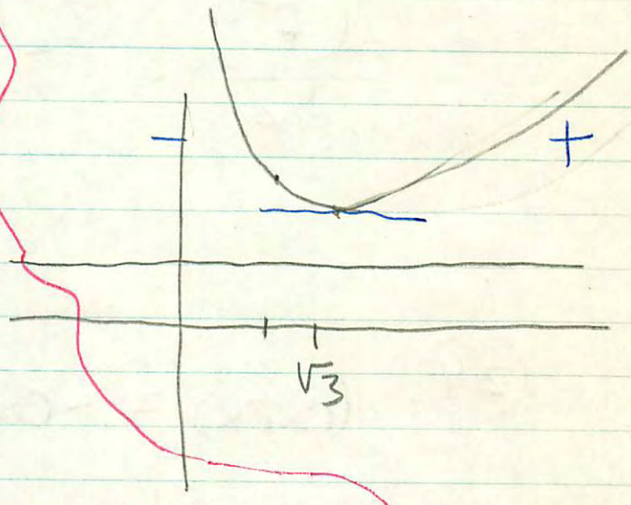
$(x - \sqrt{3})(x + \sqrt{3}) = 0$

$f' > 0 \qquad x > \sqrt{3}, \; x < -\sqrt{3}$

$f' < 0 \qquad |x| < \sqrt{3}$

$f' = 0 \qquad x \in \{\pm \sqrt{3}\}$

NOT included because



(27) ~~(28)~~

28

a) WTP $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{1}{\cos\theta} = 1 \cdot 1 = 1$$

b) $\lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h} =$

$$\lim_{h \to 0} \frac{\dfrac{\tan x + \tan h}{1 - \tan x \tan h} - \tan x}{h}$$

$$\lim_{h \to 0} \frac{\tan x + \tan h - \tan x + \tan^2 x \tan h}{h(1 - \tan x \tan h)}$$

$$\lim_{h \to 0} \frac{\tan h\,(1 - \tan^2 x)}{h\,(1 - \tan x \tan h)}$$

$\tan h \to h$, $(1 - \tan^2 x) \to \sec^2 x$, $(1 - \tan x \tan h) \to 1 - \tan x \cdot 0 = 1$

$1 \quad \sec^2 x$

Chain Rule.

5.8

pg 135-143

Chain Rule:

If f, g are differentiable fcns then $f \circ g$ is differentiable and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

"derivative of what's there times derivative of the inside"

Notice $\frac{d \ln x}{dx} = \frac{1}{x}$

$$\frac{d \ln 2x}{dx} = \frac{1}{x}$$

Let $u = 2x$

$du = 2$

$y = \ln u$

$\frac{dy}{du} = \frac{1}{u}$

$\frac{dy}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

$h(x) = f(g(x))$

$h'(x) = f'(g(x)) g'(x)$

$$\frac{d \ln nx}{dx} = \frac{1}{x}$$

§ 8

p. 142

all Routine.

1. $(2x^3-2x+5)^4$
$4(2x^3-2x+5)^3(6x^2-2)$

(2) $(\sin x)^5$
$5(\sin x)^4 \cos x$

(3) $(\sin 3x)^5$
$15(\sin 3x)^4 \cos 3x$

(4) $\cos 2x$
$-2\sin 2x$

(5) $(1+2x)^5 \cos x$
$u \quad v$
$10(1+2x)^4 \cos x + (1+2x)^5 \sin x$

(6) $\log_{10}(\sin 3x)$

$\frac{\log_e \sin 3x}{\log_e 10}$ $\quad \frac{3 \log_e 10 \cos 3x}{\sin 3x} = 3\cot 3x \ln 10$

(7) $(1+\ln 2x)^3$

$y = u^3$

$\frac{dy}{du} = 3u^2$

$\frac{3(1+\ln 2x)^2}{x}$

$u = (1+\ln 2x)$

$\frac{du}{dx} = \frac{1}{x}$

$y = 1+\ln 2x$

~~let~~ $u = 2x$

$\frac{du}{dx} = 2$

$y = 1+\ln u$

$\frac{dy}{du} = \frac{1}{u}$

$\frac{2}{2x} = \frac{1}{x}$

$\underset{u}{\frac{x}{2}}\ \underset{v}{\sqrt{4x^2+3}} + \frac{3}{4}\ln\left(2x+\sqrt{4x^2+3}\right)$

#28

$$\frac{1}{2}\sqrt{4x^2+3} + \frac{x\cdot 8x}{2\cdot 2\sqrt{4x^2+3}} + \frac{3}{4}\cdot\frac{2+\frac{8x}{2\sqrt{4x^2+3}}}{(2x+\sqrt{4x^2+3})}$$

$$\frac{4x^2+3+4x^2}{2\sqrt{4x^2+3}} + \frac{3}{4}\left[\frac{\frac{4\sqrt{4x^2+3}+8x}{2\sqrt{4x^2+3}}}{2x+\sqrt{4x^2+3}}\right]$$

$$\frac{8x^2+3}{2\sqrt{4x^2+3}} + \frac{3}{4}\left[\frac{4(2x+\sqrt{4x^2+3})}{(2x+\sqrt{4x^2+3})\,2\sqrt{4x^2+3}}\right]$$

~~$8x^2+3+6\sqrt{4x^2+3}$~~

$$\frac{8x^2+6}{2\sqrt{4x^2+3}}$$

$$\frac{4x^2+3}{\sqrt{4x^2+3}}$$

$$\sqrt{4x^2+3}$$

Since there is no real lecture for ch 5, 9

use the following.

See ~~textbook~~
ANALYSIS TEXT pg. 220
AND associated
notes!

Notation $f^{-1}(x)$ means inverse
the "-1" is applied to the fctn name
rather than to a member of the
fctn.

1:1 for each $x \rightarrow$ unique y

$f^{-1}(f(x)) = x = f(f^{-1}(x))$

Exchange of x, y

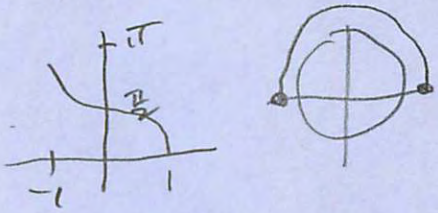
"Put z in for x, x in for y solve
for z"

Now Domain of INVERSE is obvious
and Range of original.

$A(x) = \cos^{-1} x$

$D_A \quad -1 \le x \le 1$

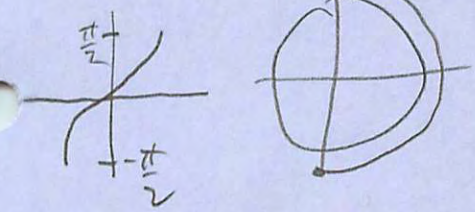
$R_A \quad 0 \le \cos^{-1} x \le \pi$



$B(x) = \sin^{-1} x$

$D_B \quad -1 \le x \le 1$

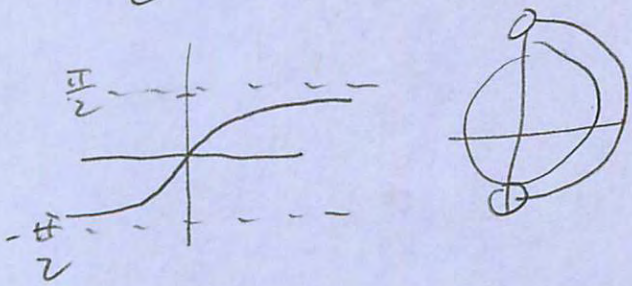
$R_B \quad -\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

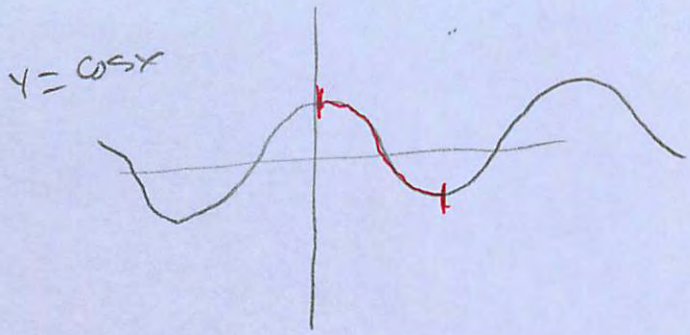


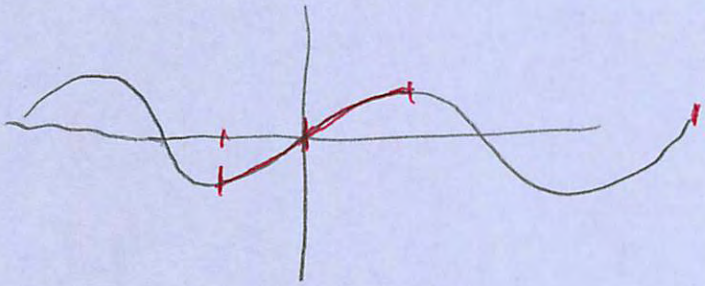
$C(x) = \tan^{-1} x$

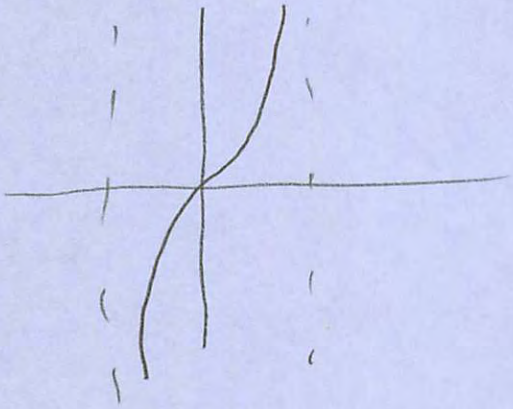
$D_C \quad x \in \mathbb{R}$

$R_C \quad -\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$





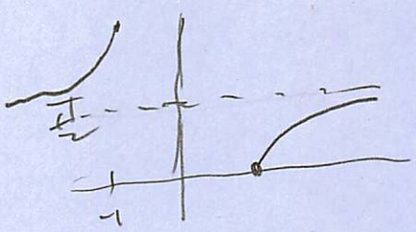


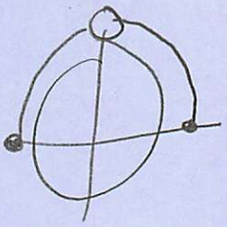


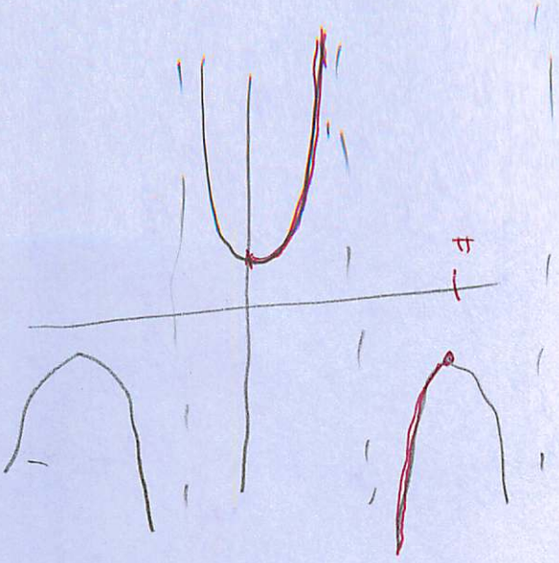
$D(x) = \sec^{-1} x$

D_D $x \le -1$ or $x \ge 1$

R_D $0 \le \sec^{-1} x \le \pi$; $\sec^{-1} x \neq \frac{\pi}{2}$



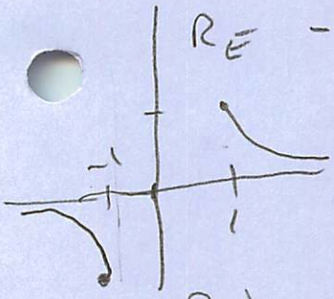


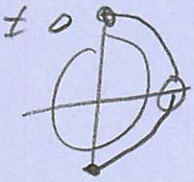


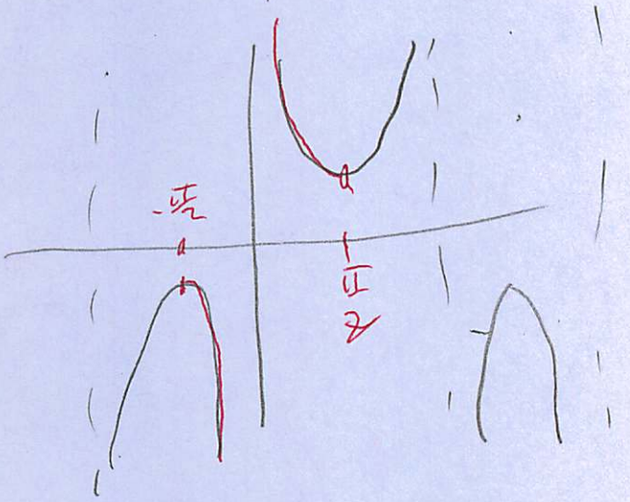
$E(x) = \csc^{-1} x$

D_E $x \le -1$ or $x \ge 1$

R_E $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$; $\sin^{-1} x \neq 0$



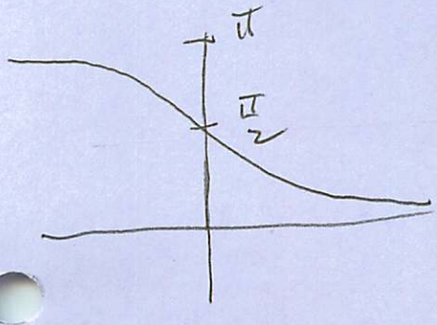


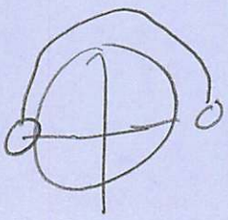


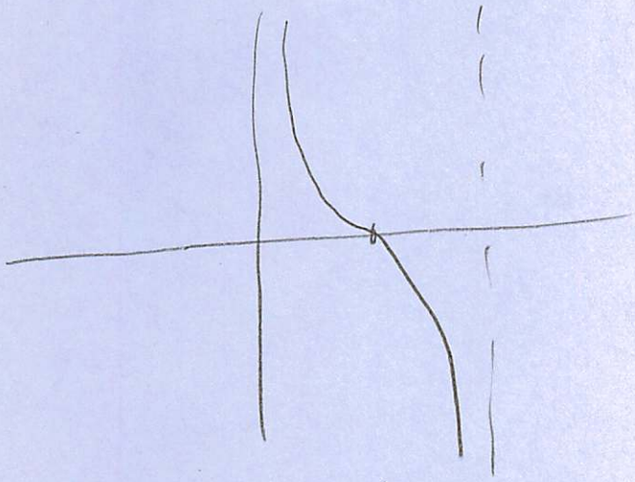
$f(x) = \cot^{-1} x$

D_f $x \in \mathbb{R}$

R_f $0 < \cot^{-1} x < \pi$



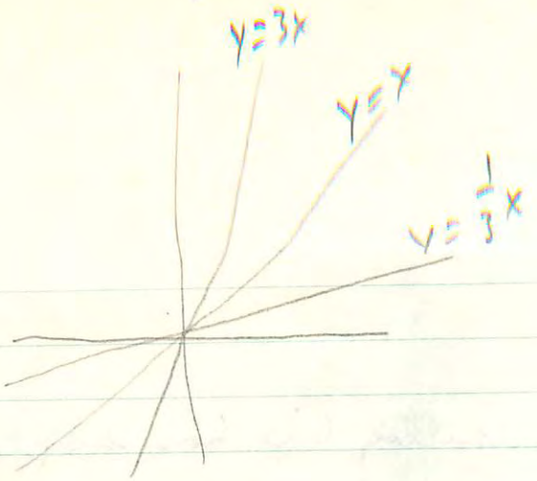




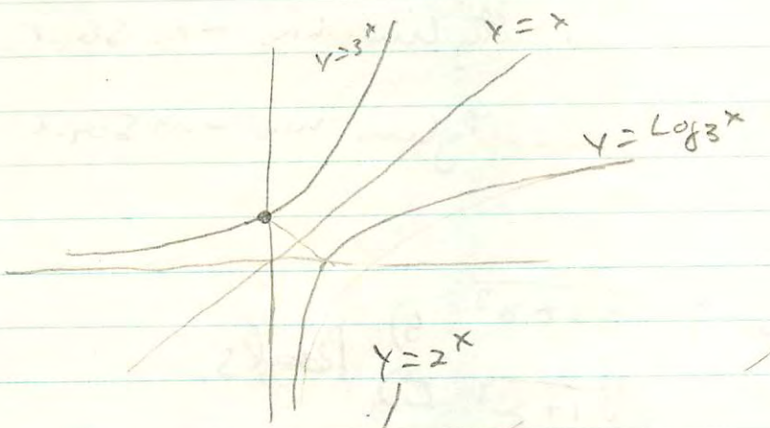
5.9

pg 143 - 147

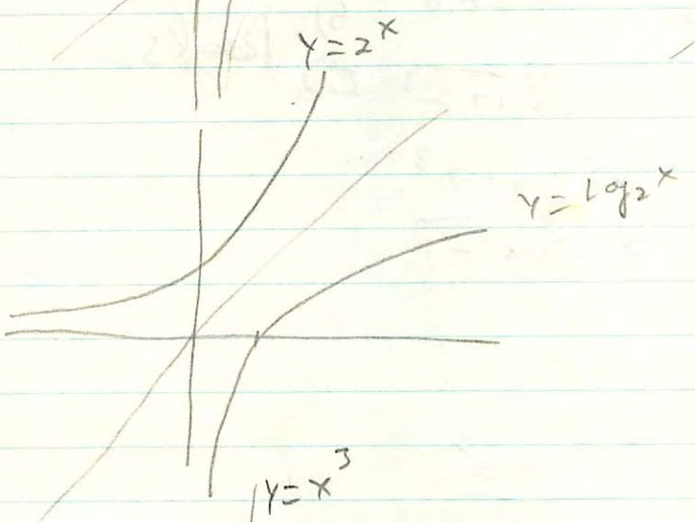
① $y = 3x$



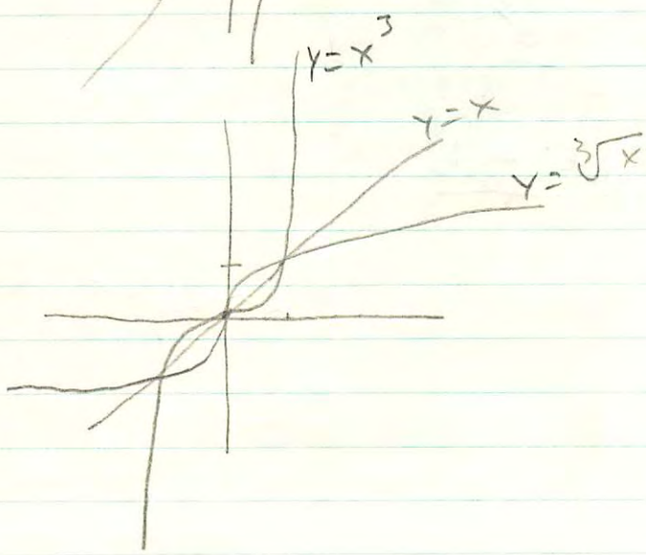
② $y = 3^x$



③ $y = \text{Log}_2 x$



④ $y = \sqrt[3]{x}$



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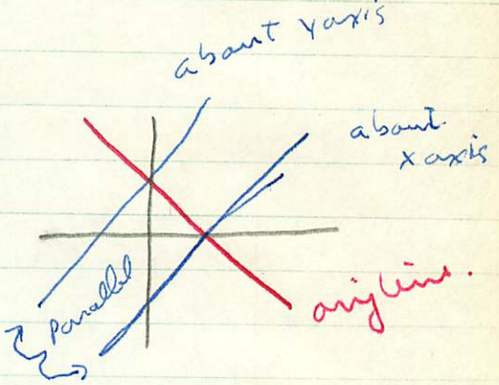
(5)

(7)

(24) a) resulting line has slope $\frac{1}{m}$

b) resulting line has $-m$ slope.

c) resulting line has $-m$ slope



(26)

a) $y = \sqrt[5]{1+x^3}$

$x = \sqrt[5]{1+z^3}$

$x^5 = 1+z^3$

$z = \sqrt[3]{x^5-1}$

b) Reals.

c)

$f(x) = e^x$

$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$

$\lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$

Deriv of e^x, e^{4x} etc.

§147 - Lecture

Thm 1 $\quad \frac{d(e^x)}{dx} = e^x$

PROOF: let $y = e^x$

$\ln y = \ln e^x$

$\ln y = x \ln e$

$\ln y = x$

using chain rule says $\frac{dx}{dy} \cdot \frac{dy}{dx} = \frac{dx}{dx} = 1$

This is really "der of inside" using chain rule

$\frac{d(\ln y)}{dy} \cdot \frac{dy}{dx} = \frac{dx}{dx}$

$\frac{1}{y} \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = y$

NOTE: we cannot use implicit diff yet so we must change to the form $\ln y$ since we know the derivative of $\ln$.

$\frac{1}{y} \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = y$ but $y = e^x$

$\therefore$

$\frac{dy}{dx} = e^x$

eg 1 $\quad \frac{d(e^{3x})}{dx}$ $\qquad$ let $u = 3x$

let $y = e^{3x}$ $\qquad \frac{du}{dx} = 3$

$\frac{dy}{du} = \frac{d(e^u)}{du} = e^u$

$\frac{du}{dx} \cdot \frac{dy}{du} = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 3e^{3x}$

In short $\quad \boxed{\frac{d(e^u)}{dx} = e^u \frac{du}{dx}}$

Lecture (cont.)

eg 2 let $y = 10^x$ find y'

START $10 = e^{\ln 10}$ then

$10^x = (e^{\ln 10})^x$ so

$y = e^{(\ln 10)x} = e^{x \ln 10}$

$\therefore \quad y' = \ln 10 \, e^{x \ln 10}$

$= \ln 10 \, e^{\ln 10^x}$

$= (\ln 10) 10^x$

~~$\ln y = x \ln 10$~~

clearly we wish to have our exponential and our logs in base e. If you are faced with the situation where the base is different then change to e!

compare w/ thm pg 371

thm 2 $\frac{d}{dx} x^a = a x^{a-1}$ $\forall a \in \mathbb{R}$.

↑ This is a change

This is not a "proof" that what we did was valid.

let $y = x^a$ note $x = e^{\ln x}$ so

$y = (e^{\ln x})^a = e^{a \ln x}$ $y' = e^a \frac{du}{dx}$ $u = a \ln x$ $du = \frac{a}{x}$

$y' = e^{a \ln x} \cdot \frac{a}{x}$

$= e^{\ln x^a} \cdot \frac{a}{x}$

$= x^a \cdot \frac{a}{x}$

$= a x^{a-1}$

Lecture cont. 5.10

eg 3 & 4 are Routine

eg 5 Let $y = x^x$ find y' ie $\left(e^{\ln x^x}\right)' =$

$x = e^{\ln x}$ so $y = e^{x \ln x}$ Now

$y' = e^{x\ln x} \cdot (1 + \ln x)$

$= e^{\ln x^x} \cdot (1 + \ln x)$

$= x^x + x^x \ln x$

Product Rule

$x \cdot \frac{1}{x} + \ln x =$

$1 + \ln x$

eg 6 $f(x) = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2) \quad a \neq 0$

find $f'(x)$. u v $u = \frac{e^{ax}}{a^3}$

$\frac{e^{ax}}{a^2}(a^2x^2 - 2ax + 2) + \frac{e^{ax}}{a^3}(2a^2x - 2a)$ $du = \frac{e^{ax}}{a^3} \cdot a = \frac{e^{ax}}{a^2}$

$+ \frac{e^{ax}}{a^2}(2ax - 2)$

$\frac{e^{ax}}{a^2}(a^2x^2) = x^2 e^{ax}.$

eg 7 $y = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) \quad a, b \neq 0$

u v

$y' = (a\sin bx - b\cos bx)\frac{a e^{ax}}{a^2+b^2} + \frac{e^{ax}}{a^2+b^2}(ab\cos bx + b^2 \sin bx)$

$= \frac{e^{ax}}{a^2+b^2}(a^2\sin bx - ab\cos bx + ab\cos bx + b^2 \sin bx)$

$= \frac{e^{ax}}{a^2+b^2}(a^2+b^2)(\sin bx) = e^{ax}\sin bx.$

end of Pg 147 Lecture. §1.10

Generally we have used inverse fcns as a tool for finding the derivatives.

The relationship between f & its inverse and the derivatives are given:

$$(f^{-1})' = \frac{1}{f(x)\, f'(x)}$$ Provided $f(x) f'(x) \neq$

composition of product.

$$(f^{-1})'(f(x))f'(x) = 1$$ $f = 5x^2 + 5x + 1$

assign Take Home Quiz

"Explain equation at bottom of Pg 147."

~~Supply 2 specific examples of its usage. These examples are to be different~~

~~from those~~ in Text $f(x) = x^2 + 1$

Then $f(x) = x^2 + 2$.

Be vague about equation & its origin. Let them sweat.

I will lead discussion in class next day. This way they will be prepared to make a contribution.

demonstrate your knowledge in two examples

for example #1 use $f(x) = \sqrt{x}$,

#2 use $f(x) = e^x$.

5-10
pg 152

① $y = 5e^{3x}$ $\quad y' = 15e^{3x}$

② $y = 5^x$ $\quad 5 = e^{\ln 5}$ $\quad y = e^{x \ln 5}$

$y' = \ln 5 \, e^{\ln 5^x} = 5^x \ln 5$

③ 2^{3x} $\quad 2 = e^{\ln 2}$

$y = e^{3x \ln 2}$ $\quad y' = 3 \ln 2 \, e^{\ln 2^{3x}}$

$y' = 2^{3x} 3 \ln 2$

④ $y = xe^x$

$y' = e^x + xe^x$

⑤ $y = e^{-x}$

$y' = -e^{-x}$

⑥ $y = x^2 e^{-x}$

$y' = 2xe^{-x} - x^2 e^{-x}$

↓

⑯ $y = \frac{10^x}{\ln 10}$ $\quad 10 = e^{\ln 10}$

$10^x = e^{x \ln 10}$

$y = e^{x \ln 10}$

$y' = \frac{10^x \ln 10}{\ln 10} = 10^x$

↓

Pg 152

#24 $y = x(\ln 5x)^2 - 2x\ln 5x + 2x$

$y' = (\ln 5x)^2 + \frac{2x \ln 5x \cdot 5}{5x} - 2\ln 5x - \frac{2x \cdot 5}{5x} + 2$

$= (\ln 5x)^2 + 2\ln 5x - 2\ln 5x$

$= (\ln 5x)^2$

5.10
Pg 152-153

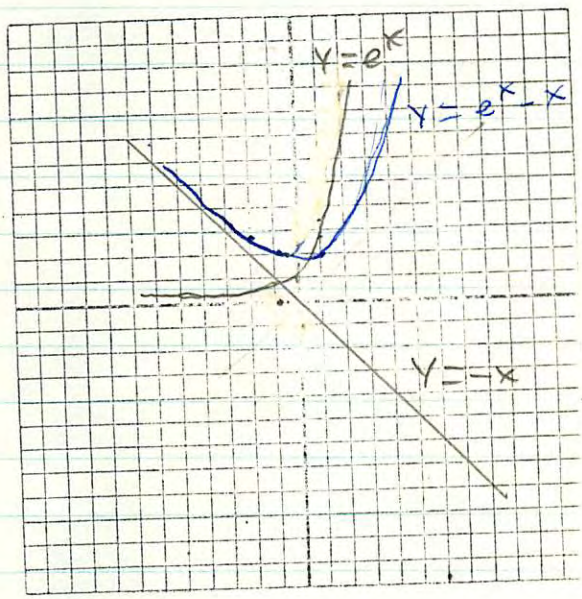
(20) $y = \ln[\ln(1+x^2)]$

$y = \ln u$ $\qquad u = \ln(1+x^2)$

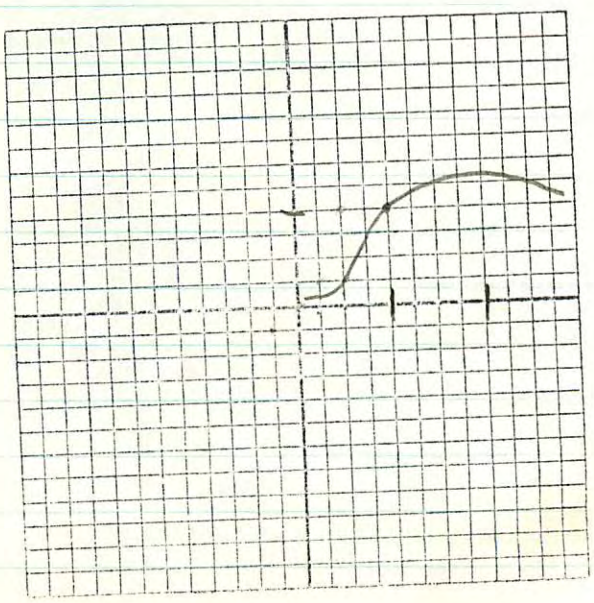
$y' = \frac{1}{u}\frac{du}{dx}$ $\qquad \frac{du}{dx} = \frac{1}{1+x^2} \cdot 2x$

$y' = \frac{1}{\ln(1+x^2)} \cdot \frac{2x}{1+x^2} = \frac{2x}{(1+x^2)\ln(1+x^2)}$

(24)



(26)



28 a) $\lim_{h\to 0} \frac{e^h - 1}{h}$ $\Rightarrow$ $\frac{e^h - 1}{h} = \frac{e^{x+h} - e^h}{h}\Big\}_{x=0}$,

ie this limit is the derivative $e^x\Big\}_{x=0}$ so

$$\frac{d(e^x)}{dx}\Big\}_{x=0} = e^x\Big\}_{x=0} = 1$$

b) $\lim_{x\to 1} \frac{2^x - 2}{x-1}$ $\Rightarrow$ let $h = x-1$

$x = h+1$

$\lim_{h\to 0} \frac{2^{h+1} - 2}{h}$ $\frac{2^{x+h} - 2^x}{h}\Big\}_{x=1}$

$2^x\Big\}_{x=1} = 2^x \ln 2\Big\}_{x=1} = 2\ln 2$

c) $\lim_{h\to 0} \frac{10^h - 1}{h}$ $\frac{10^{x+h} - 10^x}{h}\Big\}_{x=0}$

$\frac{d(10^x)}{dx}\Big\}_{x=0} = 10^x \ln 10 = \ln 10$

$a(x) = \tan x$ $\quad b(x) = \tan^{-1} x$

find $a'(x)$ and $b'(x)$

everybody knows

$a'(x) = \sec^2 x$ but nobody knows $b'(x)$.

OK ..... we'll be back to this...

Go to Next Page

If f is a 1:1 fctn then

$$f^{-1}(f(x)) = x$$

take A derivative to get

"der of the inside"

$$\left(f^{-1}(f(x))\right)' \cdot f'(x) = 1$$

This could be written:

$$f'(x) = \frac{1}{\left[f^{-1}(f(x))\right]'} \quad \text{or} \quad \left(f^{-1}(f(x))\right)' = \frac{1}{f'(x)}$$

Now Look at the bottom of Pg 147. This version seems the most Like that formula.

they have $(f^{-1})'(f(x))$ and we have $\left(f^{-1}(f(x))\right)'$.

could these items be the same? Looks promising.

Lets remove some of the disguises.

Given the fctn f and its inverse fctn g.

$$g'(f(x)) = \frac{1}{f'(x)}$$ would be a translation of

a more useful version is

$$f'(x) = \frac{1}{g'(f(x))}$$

Example 1 $f(x) = \sqrt{x}$ then $g(x) = x^2$

Now our objective is to obtain $f'(x)$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$g'(x) = 2x$

$g'(f(x)) = 2\sqrt{x}$

and, of course, it is.

Example 2 $f(x) = e^x$ then $g(x) = \ln x$

$$f'(x) = (e^x)' = \frac{1}{\frac{1}{e^x}}$$

$g'(x) = \frac{1}{x}$

$g'(e^x) = \frac{1}{e^x}$

$$= e^x$$

We didn't go through all this without a goal.

We wanted to know if given $b(x) = \tan^{-1} x$

what was $b'(x)$.

Consider: $f(x) = \tan^{-1} x$

Then $g(x) = \tan x$

$g'(x) = \sec^2 x$

$g'(f(x)) = \sec^2(\tan^{-1} x)$ so

$$f'(x) = \frac{1}{\sec^2(\tan^{-1} x)}$$

$$= \frac{1}{\frac{1}{\cos^2(\tan^{-1} x)}}$$

$$= \cos^2(\tan^{-1} x)$$

Think... let $\tan^{-1} x = u$ $\quad x \in \mathbb{R}, |u| < \frac{\pi}{2}$

$\tan u = x$

$\sin u = x \cos u$

$\sin^2 u = x^2 \cos^2 u$

$1 - \cos^2 u = x^2 \cos^2 u$

$1 = (x^2 + 1)\cos^2 u$

$\cos^2 u = \frac{1}{1+x^2}$ and

in our case $u = \tan^{-1} x$ so

$\cos^2(\tan^{-1} x) = \frac{1}{1+x^2}$

So $(\tan^{-1} x)' = \frac{1}{1+x^2}$

Generally,

$$f(\star) = \sin^{-1}\star$$

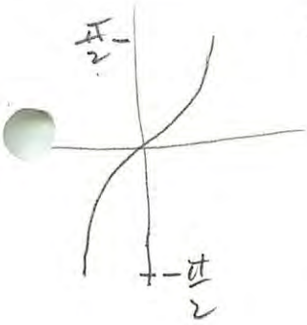
$$\text{then } f' = \frac{1}{1+\star^2}\, d\star$$

Consider $f(x) = \sin^{-1} x$

$g(x) = \sin x$

$g'(x) = \cos x$

$g'(f(x)) = \cos(\sin^{-1} x)$



notice that slope of tangent line is always positive as our result suggests.

let $u = \sin^{-1} x \quad |u| \le \frac{\pi}{2} \quad |x| \le 1$

$\sin u = \sin(\sin^{-1} x)$

$\sin u = x$

$\sin^2 u = x$

$1 - \cos^2 u = x$

$\cos^2 u = 1 - x^2$

$\cos u = \pm\sqrt{1-x^2}$ $u \in 1^{st} 4^{th}$ quad

$\cos u = \sqrt{1-x^2}$

$\cos(\sin^{-1} u) = \sqrt{1-x^2}$

So

$$f'(x) = (\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

$$f'(\star) = (\sin^{-1}\star)' = \frac{1}{\sqrt{1-\star^2}}\, d\star$$

Notice

Last Example

$f(x) = \sec^{-1} x$

$g(x) = \sec x$

$g'(x) = \sec x \tan x$

$g'(f(x)) = \sec(\sec^{-1} x)\tan(\sec^{-1} x)$

so

$$f'(x) = \sec^{-1} x = \frac{1}{\underbrace{\sec \sec^{-1} x}_{x} \tan \sec^{-1} x}$$

$$\tan \sec^{-1} x = \frac{\sin(\sec^{-1} x)}{\cos(\sec^{-1} x)}$$

This for "Remember when..."

Let $\sec^{-1} x = u$

Bottom of Pg 147. TAKE Home Quiz

$f^{-1}(f(x)) = x$ Providing f is 1:1 fctn.

To derive the formula at bottom of Pg 147

take derivs.

$\left(f^{-1}(f(x))\right) \cdot f'(x) = 1$

Der of what's there ↑ der of inside

$\left(f^{-1}(f(x))\right)' = \frac{1}{f'(x)}$

If $f'(x) = 0$ then f was a constant ∴ NOT 1:1 fctn. AND $f^{-1}(f(x)) \neq x$!

But this is too jumbled to be useable

OR

$f'(x) = \frac{1}{\left(f^{-1}(f(x))\right)'}$

More useable

Another view: f is fctn g is inverse of f.

$f'(x) = \frac{1}{g'(f)}$

Then $g(f(x)) = x$

$g'(f(x)) \cdot f'(x) = 1$

so $f'(x) = \frac{1}{g'(f(x))}$

Usage:

eg. #1

Given $f(x) = \sqrt{x}$ then $g = x^2$

$g' = 2x$

$\text{So } f'(x) = \frac{1}{2(\sqrt{x})} = \frac{1}{2\sqrt{x}}$

assuming we do not know how to take the derivative of f, we still can find the derivative of f via the inverse fctn - Providing we can find the derivative of the inverse!

The derivative of f

Take home Quiz #2

Pg #2 $f = e^x$ $g = \ln x$

$$f'(x) = \frac{1}{g'(f)} \qquad g'(x) = \frac{1}{x} \qquad g'(f) = \frac{1}{e^x}$$

$$f'(x) = \frac{1}{\frac{1}{e^x}} = e^x.$$

Go directly with:

Given $f(x) = \tan^{-1} x$ find $f'(x)$

"what? Don't Know?? Don't Know how to take the der of an inverse fctn?? oh ..... we didn't do that last year.

Consider $f(x) = \tan^{-1} x$

$g(x) = \tan x$ the inverse

$g'(x) = \sec^2 x$

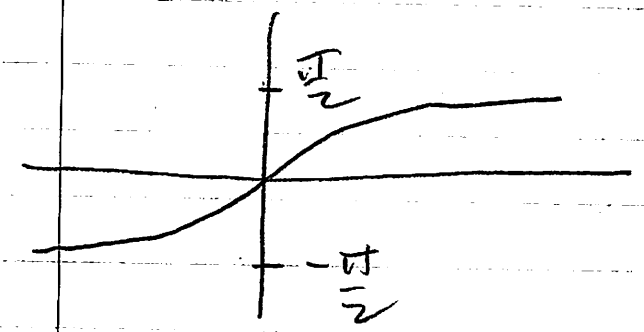
So $f'(x) = \frac{1}{g'(f)}$

$$= \frac{1}{\sec^2(\tan^{-1} x)} \equiv \cos^2(\tan^{-1} x)$$

STILL don't recognize it?

↓

$f(x) = \tan^{-1} x$



notice slope of tan line is > 0

$\forall x$

Let $\tan^{-1} x = u \quad |u| < \frac{\pi}{2} \quad x \in \mathbb{R}$.

$\tan u = x$

$\sin u = x \cos u$

$\sin^2 u = x^2 \cos^2 u$

$1 - \cos^2 u =$

$\cos^2 u (x^2 + 1) = 1$

$\cos^2 u = \frac{1}{x^2+1}$ but $u = \tan^{-1} x$ so

$\cos u = \frac{\pm 1}{\sqrt{x^2+1}}$ $u \in 1^{st} \& 4^{th}$ Quad $\therefore \cos u > 0$

Not needed since we have $\cos^2$

$\cos u = \frac{1}{\sqrt{x^2+1}}$

$\cos(\tan^{-1} x) = \frac{1}{\sqrt{x^2+1}}$

So $\cos^2(\tan^{-1} x) = \frac{1}{x^2+1}$

Thus if $f(x) = \tan^{-1} x$

Then $f'(x) = \frac{1}{x^2+1}$

$f(\bigstar) = \tan^{-1} \bigstar$

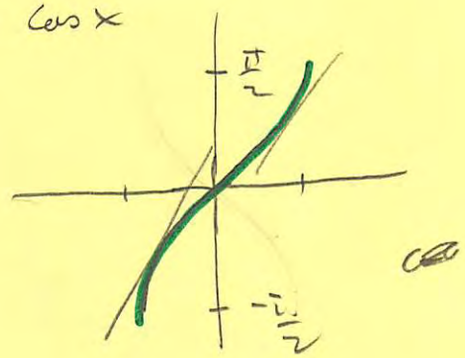
$f'(\bigstar) = \frac{1}{1+\bigstar^2} \, d\bigstar$

§ 3.11

pg 153-159

$f(x) = \sin^{-1} x$ $\quad g(x) = \sin x$

$g'(x) = \cos x$



$f'(x) = \dfrac{1}{g'(f(x))}$

$= \dfrac{1}{\cos(\sin^{-1} x)}$

Let $\sin^{-1} x = u$ $\quad |x| \le 1 \quad |u| < \frac{\pi}{2}$

$x = \sin u$

$x^2 = \sin^2 u$

$= 1 - \cos^2 u$

$\cos^2 u = 1 - x^2$

$\cos u = \pm\sqrt{1-x^2}$

Draw — u is element in quad I and IV and cosine of it is > 0

$\cos u = \sqrt{1-x^2}$

$\cos(\sin^{-1} x) = \sqrt{1-x^2}$

so

$f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

again, notice that slope of tangent line is > 0.

ie

$(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$ so $\dfrac{1}{\sqrt{1-x^2}}\, dx$

$(\sin^{-1} x)' = \sqrt{1-x^2}$

lecture
5.11
P. 156

eg 2 $y = \sin^{-1}\frac{3x}{4}$ find y'

let $u = \frac{3x}{4}$ $\frac{du}{dx} = \frac{3}{4}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ so $y = \sin^{-1} u$

$\frac{1}{\sqrt{1-u^2}} \cdot \frac{3}{4} = \frac{1}{\sqrt{1-\frac{9x^2}{16}}} \cdot \frac{3}{4}$

$\frac{3}{4\sqrt{1-\frac{9}{16}x^2}} = \frac{3}{\frac{4\sqrt{16-9x^2}}{4}} = \frac{3}{\sqrt{16-9x^2}}$

eg 3 $y = x\sqrt{1-x^2} + \sin^{-1} x$ $u = \sqrt{1-x^2}$

$\frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$

$\frac{-x}{\sqrt{1-x^2}}$

$y' = \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$

$= \frac{1-x^2-x^2+1}{\sqrt{1-x^2}}$

$= \frac{2(1-x^2)}{\sqrt{1-x^2}} = 2\sqrt{1-x^2}$

Another version.

$f(x) = \sec^{-1} x \qquad f' = ?$

$g(x) = \sec x \qquad g' = \sec x \tan x$

$$f'(x) = \frac{1}{\sec(\sec^{-1} x)\tan(\sec^{-1} x)}$$

$\hookrightarrow \quad \sec^{-1} x = \cos\left(\frac{1}{x}\right)$

$$\frac{\sin\left(\cos^{-1}\frac{1}{x}\right)}{\cos\left(\cos^{-1}\frac{1}{x}\right)} = \frac{\sqrt{1-\frac{1}{x^2}}}{\frac{1}{x}}$$

$$= \frac{\frac{\sqrt{x^2-1}}{|x|}}{\frac{1}{x}}$$

$$= \frac{x\sqrt{x^2-1}}{|x|}$$

$$\frac{1}{x \cdot \frac{x\sqrt{x^2-1}}{|x|}}$$

$$\frac{1}{\frac{x^2\sqrt{x^2-1}}{|x|}}$$

$$\frac{1}{|x|\sqrt{x^2-1}}$$

Lecture 5.11
Pg 157

This version is in text

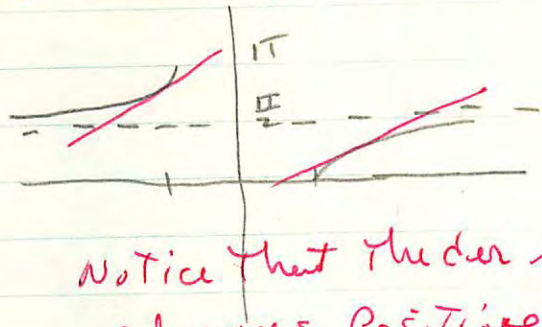
Thm $y = \sec^{-1} x$ $\quad y' = \dfrac{1}{|x|\sqrt{x^2-1}}$ $\quad |x| > 1$

That version was Pg 147.

$\sec y = x$

$1 = \dfrac{dy}{dx} \cdot \dfrac{dx}{dy}$

$\dfrac{dx}{dy} = \sec y \tan y$



$1 = \sec y \tan y \dfrac{dy}{dx}$

Notice that the der is always positive. even when $x < 0$.

$\dfrac{dy}{dx} = \dfrac{1}{\sec y \tan y} = \dfrac{1}{x \tan y}$

we still need to write $\tan y$ in terms of x.

From the picture above, when $x > 1$ $y = \sec^{-1} x$ is in the range $(0, \frac{\pi}{2})$ it 1st Quad. Tan is > 0 in 1st Quad. x is > 0 in 1st Quad so $x \tan y > 0$.

$\tan^2 y + 1 = \sec^2 y$

$= x^2$

$\tan y = \pm\sqrt{x^2-1}$

when $x < 1$ $y = \sec^{-1} x$ is in range $(\frac{\pi}{2}, \pi)$ it 2nd Quad. Tan is < 0 in 2nd Quad. x is Neg in 2nd Quad. so $x \tan y$ is > 0

Thus we need $|x|$ since $\sqrt{x^2-1}$ is always > 0, in order to drop the $\pm$ sign, so:

$$\frac{dy}{dx} = \frac{1}{x \tan y} = \frac{1}{|x|\sqrt{x^2-1}}$$

eg 4 $y = \sec^{-1} 5x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{|5x| \sqrt{25x^2 - 1}} \cdot 5$$

$$= \frac{1}{|x| \sqrt{25x^2 - 1}}$$

p159

13 $f = y = \sec^{-1} 3x$ $\quad g = \sec 3x$

$g' = 3 \sec 3x \tan 3x$

$$f' = \frac{1}{3 \sec 3(\sec^{-1} 3x) \tan 3(\sec^{-1} 3x)}$$

this is the real problem!

I can't believe we need to memorize add formulas!

$y = \sec^{-1} \star$

$$y' = \frac{1}{|\star| \sqrt{\star^2 - 1}} \, d\star$$

$$= \frac{3}{|3x| \sqrt{9x^2 - 1}}$$

$$= \frac{1}{|x| \sqrt{9x^2 - 1}}$$

(2) a) $\sin^{-1} 1 = y$ b) $\tan^{-1} 1 = y$ c) $\sin^{-1} -\frac{\sqrt{3}}{2} = y$

$y = \frac{\pi}{2}$ $y = \frac{\pi}{4}$ $y = -\frac{\pi}{3}$

d) $\tan^{-1} \sqrt{3} = y$ e) $\sec^{-1} \sqrt{2} = y$

$y = -\frac{\pi}{3}$ $y = \frac{\pi}{4}$

(3) $\sin(\tan^{-1} 1)$ $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

(4) $\tan(\sin^{-1} \frac{\sqrt{3}}{2})$ $\tan \frac{\pi}{3} = \sqrt{3}$

(5) $\tan(\sec^{-1} 2)$ $\tan \frac{\pi}{3} = \sqrt{3}$

(6) $\sin(\sin^{-1} 0.3)$ $= .3$

(7) $\tan[\sin^{-1}(-\frac{\sqrt{2}}{2})]$ $\tan -\frac{\pi}{4} = -1$

(8) $\sin \tan^{-1} 0$ $\sin 0 = 0$

(9) $y = \sin^{-1} 5x$ $y' = \frac{5}{\sqrt{1-25x^2}}$

(10) $y = \sin^{-1} e^{-x}$ $y' = \frac{-e^{-x}}{\sqrt{1-e^{-2x}}}$

$u = e^{-x}$ $\frac{du}{dx} = -e^{-x}$

(11) $\tan^{-1} 3x = y$ $y' = \frac{3}{1+9x^2}$

(12) $\tan^{-1} \sqrt[3]{x} = y$ $y' = \frac{1}{3x^{2/3}(1+x^{2/3})}$

$u = x^{\frac{1}{3}}$ $\frac{du}{dx} = \frac{1}{3}x^{-2/3}$

(13) $\sec^{-1} 3x = y$ $y = \frac{3}{3|x|\sqrt{9x^2-1}} = \frac{1}{|x|\sqrt{9x^2-1}}$

(14) $-\frac{1}{3}\sin^{-1}\frac{3}{x}$ $\left(-\frac{1}{3}\right)\left(\frac{-3}{x^2}\right)\frac{1}{\sqrt{1-\frac{9}{x^2}}}$

$u = \frac{3}{x} = 3x^{-1}$ $\frac{du}{dx} = \frac{-3}{x^2}$

$\frac{1}{x^2} \cdot \frac{1}{\frac{\sqrt{x^2-9}}{x}} = \frac{1}{|x|\sqrt{x^2-9}}$

~~Absolute value needed because $d(\sin^{-1})$ is always >0.~~

This is $-\frac{1}{3}\sin^{-1}$ so ...

(15) $y = \frac{x}{2}\sqrt{2-x^2} + \sin^{-1}\frac{x}{\sqrt{2}}$

$y' = \frac{1}{2}\sqrt{2-x^2} - \frac{x^2}{2\sqrt{2-x^2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{1-\frac{x^2}{2}}}$

$= \frac{2-x^2-x^2}{2\sqrt{2-x^2}} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2-x^2}}$

$= \frac{2-x^2-x+2}{2\sqrt{2-x^2}} = \frac{2(2-x^2)}{2\sqrt{2-x^2}} = \sqrt{2-x^2}$

$u = \sqrt{2-x^2}$

$\frac{du}{dx} = -2x \cdot \frac{1}{2\sqrt{2-x^2}}$

$\frac{-x}{\sqrt{2-x^2}}$

(16) $\sqrt{3x^2-1} - \tan^{-1}\sqrt{3x^2-1}$

$\frac{3x}{\sqrt{3x^2-1}} - \frac{1}{1+(3x^2-1)}$

$\frac{3x + 3x(3x^2-1) - \sqrt{3x^2-1}}{\sqrt{3x^2-1}\,(1+(3x^2-1))} = \frac{9x^3 - \sqrt{3x^2-1}}{}$

$u = \sqrt{3x^2-1}$

$\frac{dy}{dx} = \frac{1}{2}\frac{6x}{\sqrt{3x^2-1}}$

(17) $\frac{2}{5}\sec^{-1}\sqrt{3x^5}$

$\frac{2}{5} \cdot \frac{5}{2}\sqrt{3x^3} \cdot \frac{1}{\sqrt{3x^5}\sqrt{3x^5-1}}$

$\frac{1}{|x|\sqrt{3x^5-1}}$

absol value not needed since $x \geq 0$ as per original problem

$u = \sqrt{3x^5} = 3^{\frac{1}{2}}x^{\frac{5}{2}}$

$\frac{dy}{dx} = \frac{5}{2}3^{\frac{1}{2}} \cdot x^{\frac{3}{2}}$

$= \frac{5}{2}\sqrt{3x^3}$

domain of $\sec^{-1}x$
$0 \leq x \leq \pi$

21 a) $\ln\left(x+\sqrt{x^2-9}\right)$

$u = \sqrt{x^2-9}$

$\frac{dy}{dx} = \frac{1}{2}\frac{2x}{\sqrt{x^2-9}}$

$= \frac{x}{\sqrt{x^2-9}}$

$\frac{1}{x+\sqrt{x^2-9}} \cdot 1 + \frac{x}{\sqrt{x^2-9}}$

$\frac{\sqrt{x^2-9}+x}{(x+\sqrt{x^2-9})\sqrt{x^2-9}} = \frac{1}{\sqrt{x^2-9}}$

b) $\sin^{-1}\frac{x}{3}$

$\frac{1}{3}\Big/\sqrt{1-\frac{x^2}{9}} = \frac{1}{\sqrt{9-x^2}}$

Review ex.

$S^2 + C^2 = 1$

$C^2 + 1 = Sec^2$

① $y = 6x^2 + 3x - 1$ $\quad y' = 12x + 3$

② $(x^2+1)\sin 2x = y$ $\quad y' = 2x\sin 2x + 2\cos 2x(x^2+1)$

③ $\frac{\sin^2 x}{\cos x} = y$

$\tan x \sec x = y$

$y' = \sec x(\sec^2 x) + \tan x \tan x \sec x$

$\sec^3 x + \tan^2 x \sec x$

$\sec x(\sec^2 x + \tan^2 x)$

$\sec x(\sec^2 x + \sec^2 + 1)$

$\sec x(2\sec^2 x + 1)$

④ $\left(\frac{x^3-2x}{x^4}\right)^2 = \left(\frac{x^2-2}{x^3}\right)^2$

$2\left(\frac{x^2-2}{x^3}\right)\cdot\frac{(6-x^2)}{x^4}$

$\frac{(2x^2-4)(6-x^2)}{x^7}$

$\frac{x^3(2x) - 3x^2(x^2-2)}{x^6}$

$\frac{2x^4 - 3x^4 + 6x^2}{x^6}$

$\frac{-x^4 + 6x^2}{x^6}$

$\frac{-x^2+6}{x^4}$

⑤ $e^{-3x}\cos 2x$

$-3e^{-3x}\cos 2x + 2e^{-3x}\sin 2x$

⑥ $(2x+1)^5$ $\quad 2\times 5(2x+1)^4 = 10x(2x+1)^4$

⑦ a) $\ln\left[x^3(1-2x)^5\right] = 3\ln x + 5\ln(1-2x)$

$\frac{3}{x} - \frac{10}{1-2x} = \frac{3-6x-10x}{x(1-2x)} = \frac{3-16x}{x(1-2x)}$

b) $\ln\sqrt[3]{\cos x} = \frac{1}{3}\ln\cos x$

$\frac{-\sin x}{3\cos x} = -\frac{1}{3}\tan x$

(8) $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ $\quad -\frac{1}{2}x^{-3/2} = -\frac{1}{2\sqrt{x^3}} = -\frac{1}{2x\sqrt{x}}$

(9) $\frac{1}{3}\tan^{-1}\frac{x}{3}$ $\quad \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{1+\frac{x^2}{3^2}} = \frac{1}{9}\cdot\frac{9}{9+x^2} = \frac{1}{9+x^2}$

(10) $\sqrt[3]{2x} = 2^{\frac{1}{3}}x^{\frac{1}{3}}$ $\quad \frac{2^{\frac{1}{3}}}{3\sqrt[3]{x^2}}$

(11) $\sqrt{\sin x}$ $\quad$ let $u = \sin x$ $\quad \frac{d(u^{\frac{1}{2}})}{dx} = \frac{1}{2\sqrt{u}}$

$\frac{dy}{dx} = \cos x$

$\frac{\cos x}{2\sqrt{\sin x}}$

(12) $\cos\sqrt{x}$ $\quad \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

(13) $\ln(x+\sqrt{x^2+1})$ $\quad \frac{2x}{\sqrt{x^2+1}}$

$\frac{1}{x+\sqrt{x^2+1}}\cdot 1+\frac{x}{\sqrt{x^2+1}}$

$\frac{\sqrt{x^2+1}+x}{(x+\sqrt{\ })(\sqrt{\ })} = \frac{1}{\sqrt{x^2+1}}$

(14) $x\sqrt{1+3x^2}$

$u = 1+3x^2$

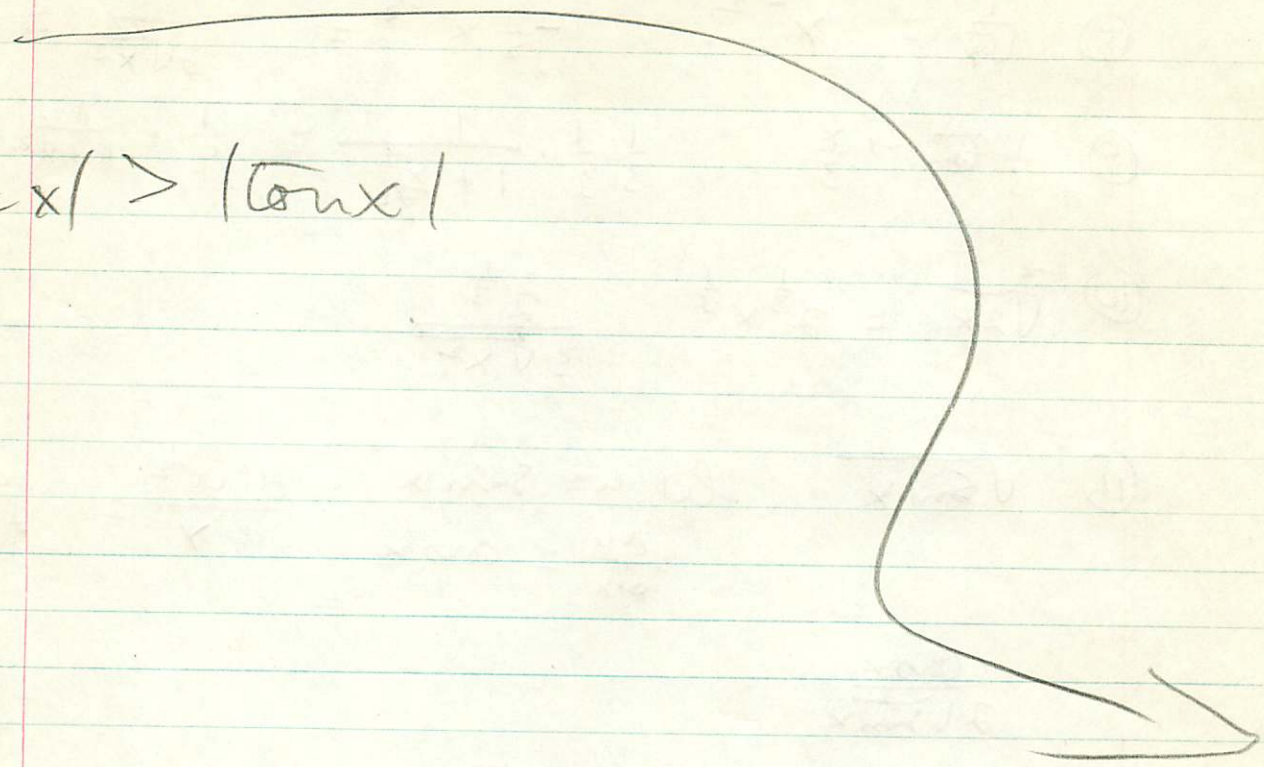
$du = 6x$

$\sqrt{1+3x^2} + \frac{3x^2}{\sqrt{1+3x^2}} =$ $\quad \frac{6x}{2\sqrt{1+3x^2}}$

$\frac{1+3x^2+3x^2}{\sqrt{1+3x^2}} = \frac{1+6x^2}{\sqrt{1+3x^2}}$

(18)

$|\sec x| > |\tan x|$



(15) $\dfrac{\sin^3 2x}{x^2+x}$ $\quad u = \sin 2x$ $\quad du = 2\cos 2x$ $\quad u^3 \quad 3u^2 \quad 3\sin^2 2x$

$$\frac{(x^2+x)\,6\cos 2x \sin^2 2x - (2x+1)\sin^3 2x}{(x^2+x)^2}$$

(16) $e^{-x}\tan x^2$

$$-e^{-x}\tan x^2 + 2x e^{-x}\sec^2 x^2$$

(17) $e^{-x}\tan^{-1} x^2$

$$-e^{-x}\tan^{-1} x^2 + \frac{2x e^{-x}}{1+x^4}$$

(18) $\ln|\sec x + \tan x|$

$$\frac{1}{|\sec x+\tan x|}\cdot \sec x\tan x + \sec^2 x$$

$$\frac{1}{|\sec x+\tan x|}\cdot |\sec x(\sec x+\tan x)| = \sec x$$

Notice: $\sec x$ is > 0. What?

(19) $(e^{x^2})^3 = e^{3x^2}$ $\quad 6x e^{3x^2}$

(20) $5^{\cos x}$ $\quad 5 = e^{\ln 5}$

$$5^{\cos x} = e^{\cos x \ln 5}$$

$$-\sin x \ln 5\, e^{\cos x \ln 5}$$

(21) $\sin^{-1} 2x$ $\quad \dfrac{2}{\sqrt{1-4x^2}}$

(22) $\csc 3x = \dfrac{1}{\sin 3x}$

$$\frac{-3\cos 3x}{\sin^2 3x} = -3\cot 3x \csc 3x$$

(23) $\log_{10}(x^2+1)$

$$\frac{\ln x^2+1}{\ln 10}$$

$$\frac{2x}{(x^2+1)\ln 10}$$

$$\frac{2x\log_{10} e}{x^2+1}$$

(50) WTS $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$ $x \neq 0$

let $y = \sec^{-1} x$ $0 \leq y \leq \pi$ and $y \neq \frac{\pi}{2}$

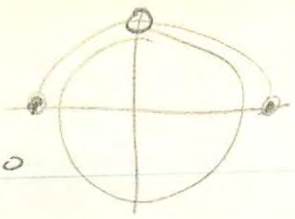
$\sec y = x$

$\frac{1}{\cos y} = x$

$\cos y = \frac{1}{x}$

$y = \cos^{-1}\left(\frac{1}{x}\right)$

(51) $f(x) = e^x + e^{-x}$ Pg 147 f^{-1}



(50) WTS $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$ $x \neq 0$

let $y = \sec^{-1} x$ $\quad 0 \leq y \leq \pi$

$\sec y = x$

$\frac{1}{\cos y} = x$

$\cos y = \frac{1}{x}$

$y = \cos^{-1}\left(\frac{1}{x}\right)$

(51) $f(x) = e^x + e^{-x}$ $\quad e^x + \frac{1}{e^x} = \frac{e^{2x}+1}{e^x}$

$x = e^z + e^{-z}$

$\ln x = \ln e^{2x+1} - \ln e^x$

$\ln x = 2x + 1 - 1$

$\ln x = 2x$

$x = \frac{e^{2z}+1}{e^z}$ $\quad \ln x = \ln(e^{2z}+1) - \ln e^z$ $\quad = ?$

Pg 147 $(f^{-1})'(f(x)) = \frac{1}{f'(x)}$ $\quad$ if $f'(x) \neq 0$

$(f^{-1})' = \frac{1}{f(x)\, f'(x)}\Big\}_{x=1}$

$= \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} = \frac{1}{e^{2x} - e^{-2x}}\Big\}_{x=1}$

$\frac{1}{e^2 - e^{-2}} = \frac{1}{e^2 - \frac{1}{e^2}} = \frac{1}{\frac{e^4 - 1}{e^2}} = \frac{e^2}{e^4 - 1}$

(52) $\lim_{x_1 \to 2} \frac{(1+x_1^2)^3 - 125}{x_1 - 2} = \lim_{x_1 \to 2} \frac{(1+x_1^2)^3 - 5^3}{x_1 - 2}$

$\lim_{x_1 \to x} \frac{(1+x_1^2)^3 - (1+x^2)^3}{x_1 - x} \Big\}_{x=2}$ $\quad f(x) = (x^2+1)^3 \Big\}_{x=2}$

$3 \cdot 2x(x^2+1)^2 \Big\}_{x=2} = 300$

(53) $\lim_{x_1 \to 3} \frac{\ln(1+2x_1) - \ln 7}{x_1 - 3} = \lim_{x_1 \to x} \frac{\ln(1+2x_1) - \ln(1+2x)}{x_1 - x} \Big\}_{x=3}$

$f(x) = \ln(1+2x)$ $\quad \frac{2}{1+2x} \Big\}_{x=3} = \frac{2}{7}$

(54) $\lim_{h \to 0} \frac{e^{(3+h)^2} - e^9}{h} = \lim_{h \to 0} \frac{e^{(x+3+h)} - e^{(x+3)^2}}{h} \Big\}_{x=0}$

$f(x) = e^{(x+3)^2}$ $\quad 2(x+3)e^{(x+3)^2} \Big\}_{x=0} = 6e^9$

$6e^9$

Another method.

$\lim_{h \to 0} \frac{e^{(x+h)^2} - e^{x^2}}{h} \Big\}_{x=3} \Rightarrow e^{x^2} \Big\}_{x=3}$ $\quad 2xe^{x^2} \Big\}_{x=3}$

$6e^9$.

(55) $\lim_{\Delta x \to 0} \frac{\sin\sqrt{3+\Delta x} - \sin\sqrt{3}}{\Delta x}$

$f(x) = \sin\sqrt{x} \Big\}_{x=3}$ $\quad \frac{1}{2\sqrt{x}} \cos\sqrt{x} \Big\}_{x=3} = \frac{1}{2\sqrt{3}} \cos\sqrt{3}$

(56) $2^{3x} \cdot 5^{7x}$ $\quad 2^{3x} = e^{3x\ln 2}$ $\quad 5^{7x} = e^{7x\ln 5}$

$3\ln 2\, e^{3x\ln 2} 5^{7x} + 7\ln 5\, 2^{3x} e^{7x\ln 5}$

$(3\ln 2) 2^{3x} 5^{7x} + 2^{3x} 5^{7x} (7\ln 5) = (3\ln 2 + 7\ln 5) 2^{3x} 5^{7x}$

continued.

(57) a) $f' = \frac{1}{x^3}$ b) $f' = \frac{1}{x^2}$ c) $f' = \frac{1}{x}$

$f(x) = \frac{3x^{2/3}}{2}$ $f = \frac{-1}{x}$ $f = \ln x$

(58) a) $f' = e^x$ b) $f' = 10^x$ $10^x = e^{x \ln 10}$

$f = e^x$ $f = \frac{10^x}{\ln 10}$ $\ln 10 \, e^{x \ln 10} = (\ln 10) 10^x$

c) $f' = \frac{1}{1+x^2}$ D) $f' = \frac{x}{1+x^2}$

$f = \tan^{-1} x$ $f = \frac{1}{2} \ln (1+x^2)$

(61) $f(x) = \tan^{-1} \sqrt{x}$ $f' = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \Big]_{x=9} = \frac{1}{10} \cdot \frac{1}{6} = \frac{1}{60}$

(62) $f(x) = \sqrt{1+4x} \Big]_{x=2}$ $4 \cdot \frac{1}{2\sqrt{1+4x}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$

(63) a) $f(1) = 2$ b) $f(3) = 3$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$ pg 165

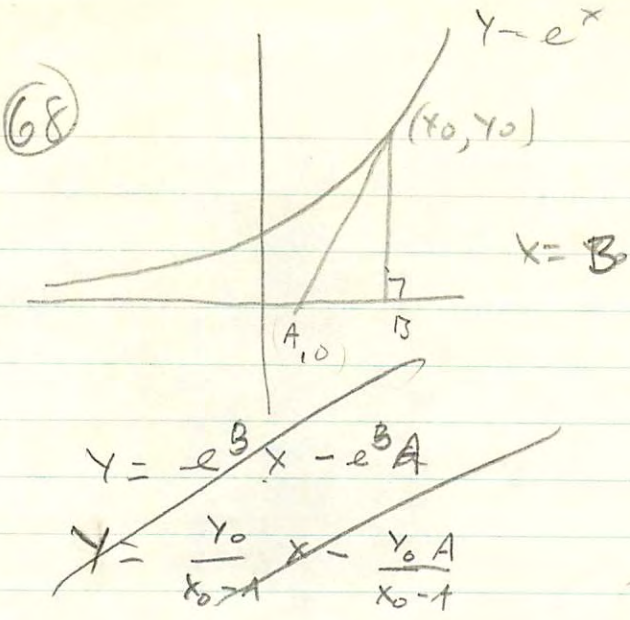
65 $f(x) = x^2 \sin\left(\frac{1}{x}\right)$

$$\lim_{h \to 0} \frac{(x+h)^2 \sin \frac{1}{x+h} - x^2 \sin \frac{1}{x}}{h} \Bigg]_{x=0}$$

$$\lim_{h \to 0} \frac{(x+h)}{h} \frac{\sin \frac{1}{x+h}}{\frac{1}{x+h}} - \frac{x}{h} \cdot \frac{\sin \frac{1}{x}}{\frac{1}{x}}$$

$$\lim_{h \to 0} \frac{x}{h} + 1 \left(\quad \right) - \frac{x}{h} \left(\quad \right)$$

(68)



$B - A = ?$

$$\frac{y_0}{x_0 - A} = e^B$$

$$(x_0, y_0) = (B, e^B)$$

$$(A, 0)$$

$$\frac{e^B}{B - A} = e^B$$

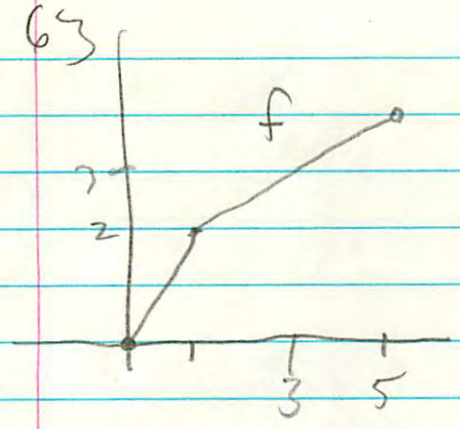
$$\frac{1}{B - A} = 1$$

$$B - A = 1$$

$$Y = e^B x - e^B A$$

$$Y = \frac{y_0}{x_0 - A} x - \frac{y_0 A}{x_0 - A}$$

63



$f(1) = 2$

$f(3) = 3$

$g(1) = \frac{1}{2}$

$g(2) = 1$

$g(3) = 3$

(66) $x \cdot x$ x^{1+1} 2^x

$x^2 = x+x+x+x+\ldots$

x terms

$2x = 1+1+\ldots$

x ones

$= 1(x)$

$2x = x$

$x^2 = x+x+x+\ldots$

x terms

fm text: is only valid if x is a positive integer

so what?

$f(x) = x^2$

$g(x) = x+x+\ldots$

x terms

$f'(3) = 6$

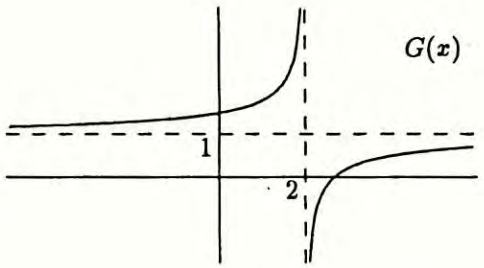
$g'(3) = 1+1+1 = 3$

"Continuous Enjoyment -- Professional Version"

#52

For each problem, select the proper multiple choice response. Include with your work a detailed explanation of your method and justification.

1) Given the function $G(x)$ as graphed on the right, which of the following statements are true?



I. $x = 2$ is a vertical asymptote of $G(x)$ because $\lim_{x \to 2^-} G(x) = +\infty$.

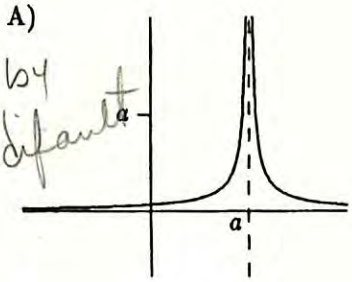
II. $x = 2$ is a vertical asymptote of $G(x)$ because $\lim_{x \to 2^-} G(x) = \lim_{x \to 2^+} G(x)$

III. $\lim_{x \to 2^+} G(x) = -\infty$

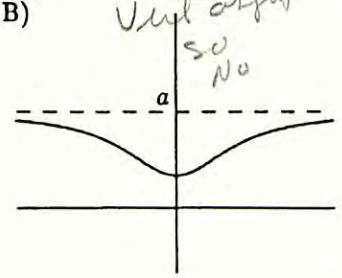
a) I only b) II only c) III only d) I and III only e) I, II, and III

2) Which is a possible graph of $f(x) = \dfrac{a}{(a-x)^2}$?

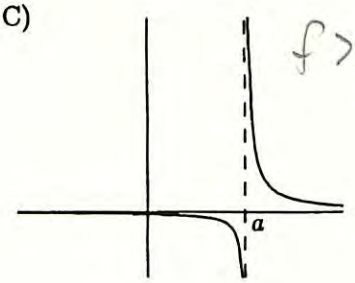
A)



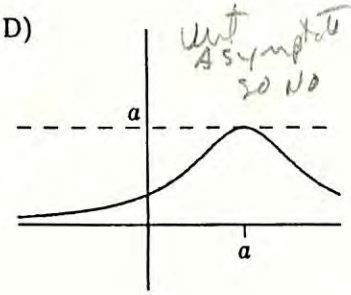
B)



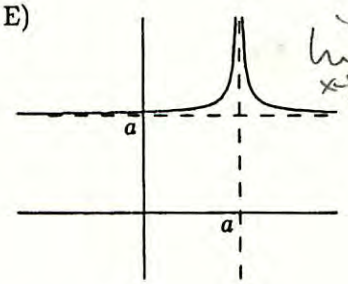
C)



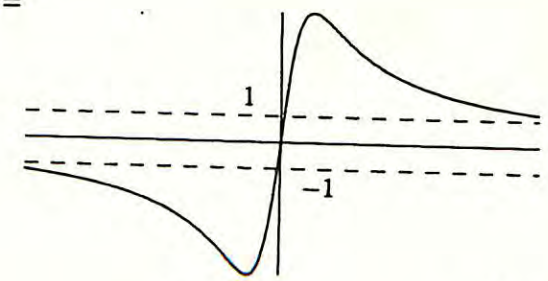
D)



E)

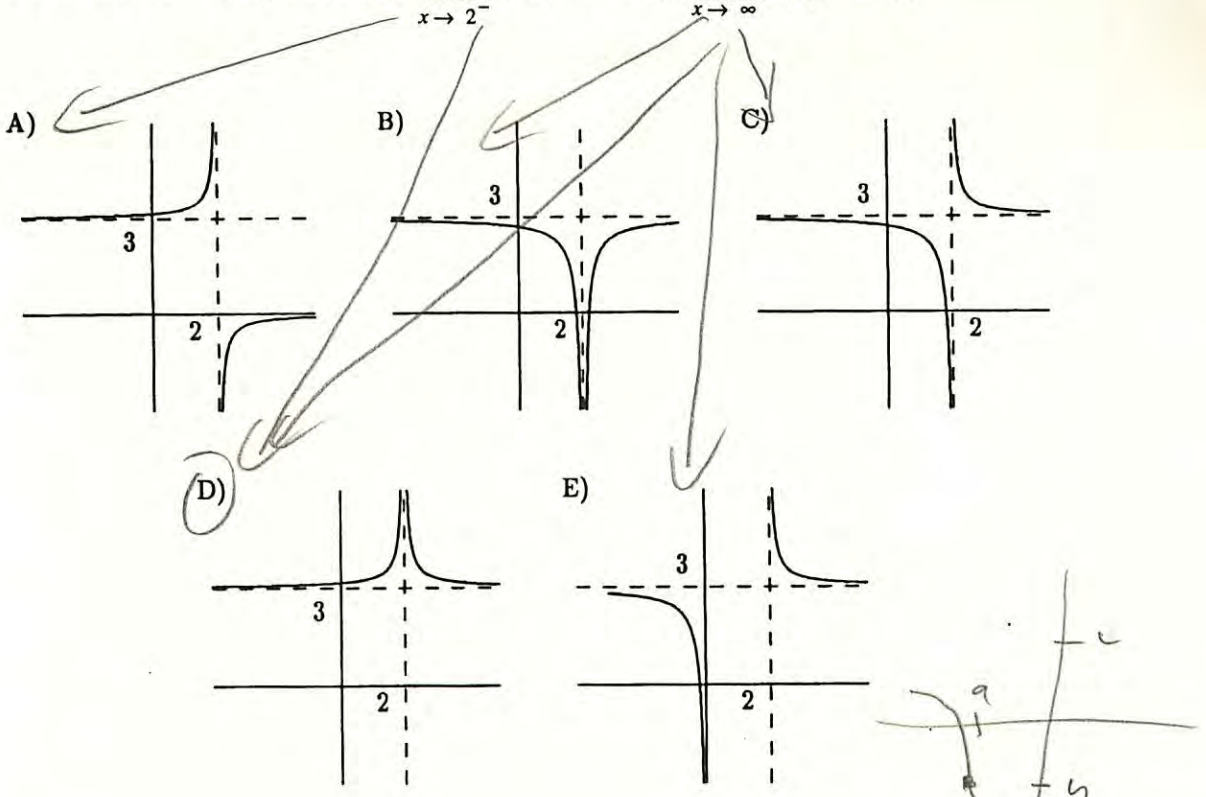


3) On the graph of $f(x)$ on the right, $y = 1$ and $y = -1$ are horizontal asymptotes. Therefore, $\lim_{x \to -\infty} f(|x|) =$



a) - 1 b) 0 c) 1 d) 2 e) Limit DNE

4) A possible graph for a function $f(x)$ such that $\lim_{x\to 2^-} f(x) = \infty$ and $\lim_{x\to \infty} f(x) = 3$ is:

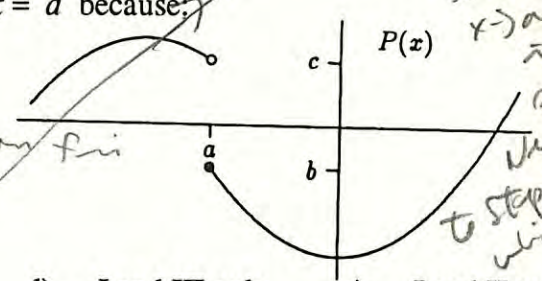


5) The function $P(a)$ graphed on the right is not continuous at $x = a$ because:

I. $\lim_{x\to a} P(a)$ does not exist

II. $P(a)$ is not defined

III. $P(a) \neq c$



a) I only b) II only c) III only d) I and III only e) I and II only

6) Which of the following functions are continuous for all real numbers?

I. $y = 2x^{4/3}$ II. $y = \sqrt[5]{3x^2 - 1}$ III. $y = \dfrac{2x+5}{2x^2+5}$

a) None B) I only c) II only d) I and II only e) All

7) Given $f(x) = \begin{cases} 3x-1 & \text{if } x < -3 \\ -x^2 - 1 & \text{if } -3 \le x \le 2 \\ 3x+1 & \text{if } x > 2 \end{cases}$ which of the following statements are true?

I. $f(x)$ is continuous at $x = -3$ II. $f(-3) = \lim_{x\to -3} f(x)$ III. $f(2) = \lim_{x\to 2} f(x)$

a) I only b) II only c) III only d) I and II only e) II and III only

8) If $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$ then $(f \circ g)(x)$ is continuous for

a) $(-\infty, \infty)$ b) $(0, \infty)$ c) $[0, \infty)$ d) $(-\infty, -2) \cup (2, \infty)$ e) $(2, \infty)$

9) Select the function below which is continuous for all real numbers except $x = 1$ and $x = -1$.

a) $f(x) = \dfrac{x^2+4}{(x-1)(x+2)}$ b) $f(x) = \dfrac{3x+1}{\sqrt{x}}$ c) $f(x) = \dfrac{2x-1}{x^2+1}$

d) $f(x) = \dfrac{2x}{x^2-1}$ e) $f(x) = \dfrac{x^2-1}{x^2+1}$

10) If $f(x) = \begin{cases} 2x-3 & \text{for } x > -1 \\ kx^3 & \text{for } x \le -1 \end{cases}$ then for $f(x)$ to be continuous everwhere, k must equal:

$\lim_{x\to -1^+} 2x-3 = \lim_{x\to -1^-} kx^3$

a) 0 b) 1 c) -1 d) 5 e) -5

11) Which of the following statements is false if $f(x) = \dfrac{1}{x}$, $g(x) = x^2$, and $a \neq 0$?

a) $\lim_{x\to a} f(g(x)) = \dfrac{1}{a^2}$ b) $\lim_{x\to a} [f(x)\cdot g(x)] = a$ c) $\lim_{x\to a} f(f(x)) = \dfrac{1}{a}$

d) $\lim_{x\to a} g(g(x)) = a^4$ e) $\lim_{x\to a} \dfrac{g(x)}{f(x)} = a^3$

$y' = 2x+6$

12) Find the point or points on the graph of $y = x^3 + 3x$ where the tangent to the graph is parrallel to $6x - y = 5$.

a) $(1, 0)$ or $(-1, 0)$ b) $(1, 4)$ c $(3, 0)$ d) $(-1, -4)$ or $(1, 4)$ e) $(1, 1)$

$(-1,-4)$ or $(1,4)$

13) $D_x\left(\sqrt{x+1}\right)$ is represented by which of the following:

a) $\lim_{\Delta x\to 0} \dfrac{\sqrt{\Delta x+1}-1}{\Delta x}$ b) $\lim_{\Delta x\to 0} \dfrac{\sqrt{x+\Delta x}-\sqrt{x+1}}{\Delta x}$ c) $\lim_{\Delta x\to 0} \dfrac{\sqrt{x+\Delta x+1}-\sqrt{x+1}}{\Delta x}$

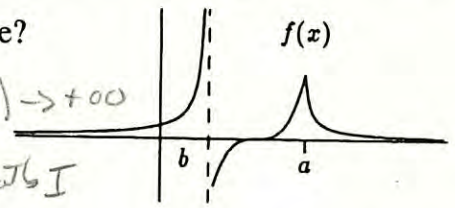
2nd def of der

d) $\lim_{\Delta x\to 0} \dfrac{\sqrt{x+\Delta x+1}-\sqrt{\Delta x+1}}{\Delta x}$ e) $\lim_{\Delta x\to 0} \dfrac{\sqrt{x+\Delta x+1}-\sqrt{x+\Delta x}}{\Delta x}$

14) Find the equation of the line perpendicular to $y = \sqrt{x+2}$ when $x = 2$.

a) $y = -4x + 8$ b) $x - 4y = -6$ c) $4x + y = 6$ d) $x + 4y = 10$ e) $4x + y = 10$

15) Given the function graphed on the right, which of the following is true?



I. $f(x)$ is not continuous at $x = b$. True

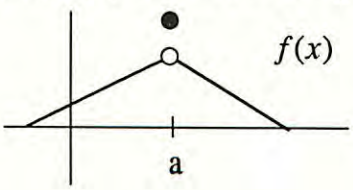
II. $f'(a^+) \neq f'(a^-)$. True $f'(a^+) \to -\infty$ $f'(a^+) \to +\infty$

III. $f(x)$ is not differentiable at $x = b$. True Find cont of I

should have been $x = a$

a) I only b) II only c) III only d) II and III only e) I, II, and III

16) Which of the following statements is true about the graph of $f(x)$ shown on the right?



I. $f(x)$ exists at $x = a$.

II. $f(x)$ is continuous at $x = a$. NO

III. $f(x)$ is differentiable at $x = a$. NO

a) I only b) II only c) III only d) I and II only e) I, II, and III

17) If $f(x) = (x^2+4)(3x^3-2)$, then $\frac{df}{dx} =$

a) $18x^3$ b) $15x^4+36x^2-4x$ c) $12x^4+36x^2+4x$

d) $18x^4+36x^2-4x$ e) $18x^3+36x^2-4x$

18) If $f(x) = x^{5/3} - x^{2/3}$ for what values of x will $f(x)$ have horizontal tangents?

a) 3 b) 2 c) $\frac{2}{3}$ d) $\frac{3}{5}$ e) $\frac{2}{5}$

19) Find an equation of a line tangent to $y = x^4 - 8x$ that is parallel to $y + 4x = 2$.

a) $y = -7$ b) $x = 1$ c) $4x + y = 29$ d) $x + 4y = 29$ e) $4x + y = -3$

20) Given $f(x) = \frac{2}{x} - \frac{3}{x^2} + \frac{5}{3x^3}$. Find $f'(x)$.

a) $2 + \frac{6}{x^3} - \frac{5}{x^4}$ b) $-\frac{2}{x^2} + \frac{6}{x^3} - \frac{5}{x^4}$ c) $-\frac{2}{x^2} - \frac{6}{x^3} - \frac{5}{x^4}$ d) $-\frac{2}{x^2} + \frac{6}{x^3} + \frac{5}{x^4}$ e) $-\frac{2}{x^2} - \frac{6}{x^3} + \frac{5}{x^4}$

21) Given $y = \frac{1-3x}{2x-5}$. Find $\frac{dy}{dx}$.

a) $-\frac{3}{2}$ b) $\frac{-12x+17}{(2x-5)^2}$ c) $\frac{13}{(2x-5)^2}$ d) $\frac{-17}{(2x-5)^2}$ e) $\frac{-3}{(2x-5)^2}$

22) Find $D_x\left[\frac{x^2-5x+6}{x-3}\right]$

a) $2x-5$ b) $\frac{3x^2-16x+21}{(x-3)^2}$ c) $-\frac{3x^2-16x+21}{(x-3)^2}$ d) $5x-2$ e) 1

23) If $f(x) = \frac{x^2+2x}{x-1}$, then $f'(0) =$

a) 2 b) 6 c) 14 d) -2 e) 3

24) Find the derivative of the function $f(x) = \frac{x^2}{3} - \frac{3}{x^2}$.

a) $\frac{4x^2-9}{6x}$ b) $\frac{2x^4+18}{3x^3}$ c) $\frac{x^3-9}{3x}$ d) $\frac{2x^4-18}{3x^3}$ e) $\frac{18-2x^4}{3x^3}$

25) If $f(x) = \sqrt{x-\sqrt{x}}$, then $f'(x) =$

a) $\frac{1}{2\sqrt{x}\sqrt{x-\sqrt{x}}}$ b) $\frac{\sqrt{x}}{2\sqrt{x}\sqrt{x-\sqrt{x}}}$ c) $\frac{2\sqrt{x}-1}{4\sqrt{x}\sqrt{x-\sqrt{x}}}$ d) $\frac{1}{\sqrt{x-\sqrt{x}}}$ e) $\frac{\sqrt{x-\sqrt{x}}}{2(2\sqrt{x}-1)}$

26) The equation of the horizontal tangent line to $f(x)=\frac{x^2}{2}+x-1$ is

a) $y=-1$ b) $y=0$ c) $y=-\frac{3}{2}$ d) $y=-\frac{1}{2}$ e) $y=-\frac{3}{8}$

27) A particle is moving along a straight line according to the equation $s=-3\cos 2t$ where $0\le t\le 2\pi$. Find the intervals of time when the particle is moving to the left.

a) $0<t<\frac{\pi}{2}$ b) $\frac{\pi}{2}<t<\frac{3\pi}{2}$ c) $\pi<t<2\pi$ d) $0<t<\pi$ e) $\frac{\pi}{2}<t<\pi$ or $\frac{3\pi}{2}<t<2\pi$

28) If $f(x)=6\sin 2x\cos 2x$ then $f'\left(\frac{\pi}{2}\right)=$

a) 0 b) 3 c) 6 d) 12 e) -3

29) The slope of the normal line to the curve $f(x)=2\cos 4x$ at $\frac{\pi}{12}$ is:

a) $-4\sqrt{3}$ b) $\frac{\sqrt{3}}{12}$ c) -4 d) $\frac{1}{4}$ e) $4\sqrt{3}$

30) If $f(x)=\frac{1+\sin x}{\sin x-1}$, then $f'(x)=$

a) $\frac{2\sin x\cos x}{(\sin x-1)^2}$ b) $\frac{2\cos x}{(\sin x-1)^2}$ c) $\frac{2}{\cos x}$ d) $\frac{-2\cos x}{(\sin x-1)^2}$ e) 1

31) If $f(x)=\tan x+\sec x$, then $f'\left(\frac{\pi}{6}\right)=$

a) 2 b) $4+2\sqrt{3}$ c) $\sqrt{3}$ d) $2+\sqrt{3}$ e) $\frac{\sqrt{3}}{3}$

32) If $s(t)=\sin t\cos t$ describes the position of a particle moving on a straight line for time $t>0$, what is the first time when the velocity of the particle is zero?

a) $\frac{\pi}{8}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) $\frac{\pi}{2}$

33) The derivative of $f(x)=\left(x^2-4x+4\right)^{35}$ is:

a) $2x-4$ b) $\left(x^2-4x+4\right)^{34}$ c) $(2x-4)\left(x^2-4x+4\right)^{34}$ d) $70(x-2)^{69}$ e) none of these

34) The equation of the line tangent to $f(x)=\left(3x^2-2x+1\right)$ at $x=0$ is:

a) $y=-2x+1$ b) $y=1$ c) $x=0$ d) $y=-\frac{1}{2}x+1$ e) $y=\frac{1}{2}x+1$

35) A particle moves in a straight line according to the equation $s(t)=\left(3t^2-2t\right)^5$. Find the instaneous velocity of the particle at time $t=1$.

a) 1 b) 4 c) 20 d) 5 e) 0

36) The derivative of $\tan^3 2x$ is:

a) $\left(\sec^2 2x\right)^3$ b) $3\tan^2 2x$ c) $3\tan^2 2x\sec^2 2x$ d) $6\tan^2 2x\sec^2 2x$ e) $3(\sec 2x)^2$

37) Find the derivative of $\left(x^2+1\right)^{-3}$

a) $-6x\left(x^2+1\right)^{-2}$ (b) $-6x\left(x^2+1\right)^{-4}$ c) $-3\left(x^2+1\right)^{-4}$ d) $-3\left(x^2+1\right)^{-2}$ e) $-3(2x)^{-4}$

38) $\lim_{h\to 0}\dfrac{\tan^2\left(\frac{\pi}{4}+h\right)-\tan^2\frac{\pi}{4}}{h}=$ Def of Der of $\tan^2 x$ at $\frac{\pi}{4}$

a) 2 b) $2\sqrt{2}$ (c) 4 d) 8 e) does not exist

39) Let $y=\sqrt[3]{\left(x^2-6x+9\right)^2}$, then $\dfrac{dy}{dx}=$

a) $(x-3)^{4/3}$ (b) $\dfrac{4}{3}(x-3)^{1/3}$ c) $\dfrac{2}{3\left(x^2-6x+9\right)^{1/3}}$ d) $\dfrac{4(x-3)}{3}$ e) $\dfrac{4(x-3)\left(x^2-6x+9\right)^{1/3}}{3}$

40) Which of the following functions are not differentiable at $x=0$?

I. $f(x)=3x^{\frac{2}{3}}$ II. $f(x)=5x^{\frac{6}{5}}$ III. $f(x)=\left(x^2-1\right)^{\frac{3}{2}}$

a) I only b) II only c) III only (d) II and III only e) I, II, and III

I

41) $\dfrac{d}{dx}\left(2\sqrt{x}-\dfrac{3}{\sqrt{x}}+\dfrac{3}{\sqrt[3]{x}}\right)$ evaluated at $x=1$ is:

) 2 b) $-\dfrac{3}{2}$ (c) $\dfrac{3}{2}$ d) -2 e) 4

42) $f(x)=\sin\sqrt{2x}$, then $f'(x)=$

a) $\cos\sqrt{2x}$ (b) $\dfrac{\cos\sqrt{2x}}{\sqrt{2x}}$ c) $\dfrac{2\cos\sqrt{2x}}{\sqrt{2x}}$ d) $\dfrac{\sin\sqrt{2x}}{\sqrt{2x}}$ e) $\dfrac{-\cos\sqrt{2x}}{\sqrt{2x}}$

43) $D_t\sqrt{\dfrac{t^2+1}{2t-3}}=$ u v

(a) $\sqrt{\dfrac{2t-3}{t^2+1}}\left(\dfrac{t^2-3t-1}{(2t-3)^2}\right)$ b) $\sqrt{\dfrac{2t-3}{t^2+1}}\left(\dfrac{3t^2-3t+2}{(2t-3)^2}\right)$ c) $\sqrt{\dfrac{t^2+1}{2t-3}}\left(\dfrac{t^2-3t-1}{(2t-3)^2}\right)$

d) $\dfrac{1}{2}\sqrt{\dfrac{2t-3}{t^2+1}}$ e) $\sqrt{\dfrac{2t+1}{2}}$

44) If $f(1)=4$ and $f'(1)=3$ and $g(x)=\sqrt{f(x)}$, then $g'(1)=$

$g'(x)=\dfrac{1}{2\sqrt{f(x)}}\cdot f'(x)$

a) $\dfrac{1}{4}$ (b) $\dfrac{3}{4}$ c) $\dfrac{\sqrt{3}}{2}$ d) $\dfrac{3}{2}$ $\frac{1}{2\sqrt{4}}\cdot 3$ e) 2

45) If $y=\cos^2 x-\sin^2 x$ then $\dfrac{d^2y}{dx^2}=$

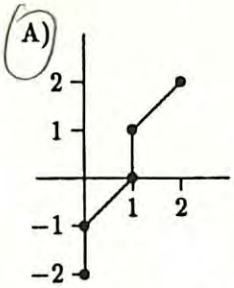
$=\cos 2x$

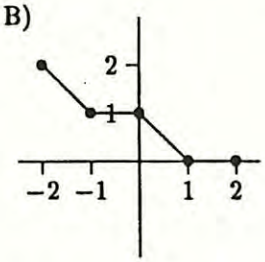
a) 0 b) $4\cos x\sin x$ c) $-2\sin 2x$ d) $4\sin 2x$ (e) $-4\cos 2x$

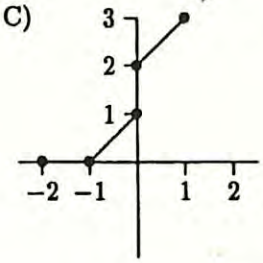
$y'=-2\sin 2x$

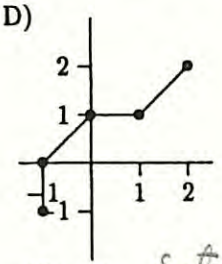
$y''=-4\cos 2x$

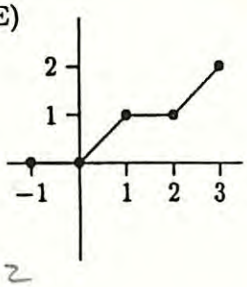
then the graph of the inverse relation of $f(x)$ is:

A)


B)


C)


D)


E)


47. The value of k for which $f(x)=\dfrac{x-2}{x+k}$ is its own inverse is:

a) 2 b) -2 c) -1 d) 1 e) 0

48. Which of the following functions are their own inverses?

I. $f(x)=\dfrac{1}{x}$ II. $f(x)=\dfrac{1}{x^3}$ III. $f(x)=x$

a) I only b) II only c) III only d) I and III only e) II and III only

49. If $f(x)=\dfrac{x+1}{x-2}$ then $f^{-1}(x)=$

a) $\dfrac{x-2}{x+1}$ b) $\dfrac{2x+1}{x-1}$ c) $\dfrac{2x-1}{x+1}$ d) $\dfrac{x-1}{2x+1}$ e) There is no inverse

50. Which of the following functions are one-to-one?

I. $f(x)=x^3-x^2$ II. $f(x)=x^3-2x$ III. $f(x)=x^3+2$

a) I only b) II only c) III only d) I and III only e) II and III only

51. If $f(x)=\dfrac{1}{x}$ and $f(g(x))=x$ then $g(x)=$

a) x b) x^2 c) $\dfrac{1}{x}$ d) $\dfrac{1}{x^2}$ e) $-x$

52. If $f(x_0)=2$, $f'(x_0)=3$, and $f(x)$ is an increasing function, find the derivative of f^{-1} evaluated at 2.

a) $\dfrac{1}{2}$ b) 2 c) 3 d) $-\dfrac{1}{3}$ e) $\dfrac{1}{3}$

53. If $f(x)=\sqrt[3]{x}$, then find the function which is the derivative of the inverse of f.

a) $\dfrac{\sqrt[3]{x}}{3x}$ b) $3x^2$ c) $2x$ d) $\sqrt{3x}$ e) x^3

54. $\log_{e^2} e=$

a) 0 b) $\dfrac{1}{2}$ c) 1 d) 2 e) 4

55. If $f(x)$ is a one-to-one continuous function with the values given in the table on the right, then for the inverse function, f^{-1}, the derivative of f^{-1} at $x = 2$ is :

x	$f(x)$	$f'(x)$
1	2	4
2	5	6

$g'(2) = \frac{1}{f'(g(2))}$

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{5}{6}$ e) $-\frac{1}{25}$

$f(\text{what}) = 2$ what $= 1$ so $\frac{1}{f'(1)} = \frac{1}{4}$

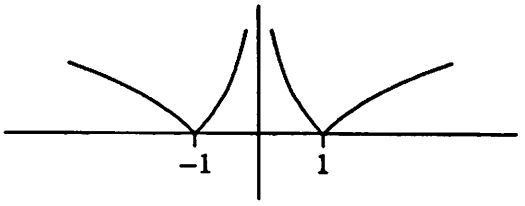
56. For what values of x does $\ln x^4 = 4 \ln x$?

a) all real numbers b) $x \geq 0$ c) $x > 0$ d) $x \geq 1$ e) $x \geq e$

57. If $f(x) = \sqrt{\ln x^3}$, then $f'(x) =$

a) $\frac{3}{2x}$ b) $\frac{1}{x\sqrt{x}}$ c) $\frac{\sqrt{3}}{2x\sqrt{\ln x}}$ d) $\frac{\sqrt{3}}{2\sqrt{\ln x}}$ e) $\sqrt{\frac{3}{\ln x}}$

58. The graph show on the right is most like:



a) $f(x) = |\ln x|$ b) $f(x) = \frac{1}{|\ln x|}$ c) $f(x) = \ln|x|$ d) $f(x) = \frac{1}{\ln|x|}$ e) $f(x) = |\ln|x||$

59. If $f(x) = \ln(\cos 2x)$, then $f'(x) =$

a) $\tan 2x$ b) $\cot 2x$ c) $-2 \tan 2x$ d) $-2 \cot 2x$ e) $2 \cot 2x$

60. $\frac{d}{dx} \ln(3 - 2x) =$

a) $\frac{1}{3-2x}$ b) $\frac{2}{3-2x}$ c) $\frac{-2}{3-2x}$ d) $\frac{-1}{3-2x}$ e) $\frac{3}{3-2x}$

61. The equation of the line tangent to $y = \ln x$ at $x = e^2$ is:

a) $y = \frac{1}{e^2}x + 1$ b) $y = \frac{1}{e^2}x$ c) $y = e^2x - 2$ d) $y = e^2x + 2$ e) $y = \frac{1}{e^2}x - 3$

62. If $f(x) = x \ln x$, then $f'(x) = 0$ when $x =$

a) 1 b) 0 c) e d) e^2 e) $\frac{1}{e}$

63. $\lim_{x \to 0} \frac{\ln(2+x) - \ln 2}{x} =$ Def of der of $\ln x$ at $x = 2$

a) $\ln \frac{1}{2}$ b) 0 c) $\frac{1}{2}$ d) $\ln 2$ e) e^2

64. $\frac{d}{dx}\left[\sin e^{2x}\right] =$

a) $e^{2x} \cos e^{2x}$ b) $2e^{2x} \cos e^{2x}$ c) $\cos e^{2x}$ d) $-2x \cos e^{2x}$ e) $-e^{2x} \cos e^{2x}$

65. $\lim_{x \to \infty} \tan^{-1} x =$

a) ∞ b) $-\infty$ c) 0 d) $\frac{\pi}{2}$ e) $-\frac{\pi}{2}$

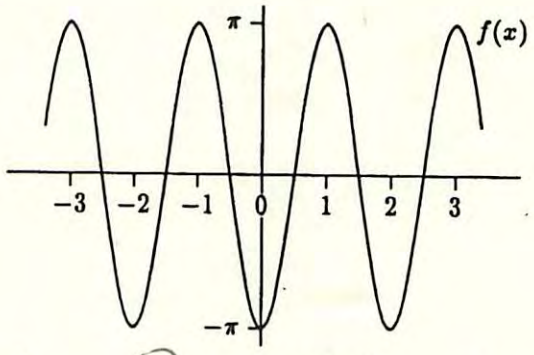
66. Find $\tan\left(\sin^{-1}\left(\frac{2}{3}\right) + \csc^{-1} 3\right)$

a) $\frac{1}{2\sqrt{2}} - \frac{2}{\sqrt{5}}$ b) $\frac{1}{2\sqrt{2}} + \frac{2}{\sqrt{5}}$ c) $\frac{4\sqrt{2}+\sqrt{5}}{2\sqrt{10}+1}$ d) $\frac{4\sqrt{2}+\sqrt{5}}{2\sqrt{10}-1}$ e) $\frac{3}{\sqrt{5}+2\sqrt{3}}$

67. If $\tan\theta = 2$, find $\tan\left(\frac{\pi}{3}+\theta\right)$.

a) $2+\sqrt{3}$ b) $2+\frac{\sqrt{3}}{3}$ c) $\frac{2+\sqrt{3}}{1-2\sqrt{3}}$ d) $\frac{2+\sqrt{3}}{1+2\sqrt{3}}$ e) $\frac{1+\sqrt{3}}{2}$

68. The graph of $f(x)$ shown on the right is a variation of the cox x graph. Therefore, $f(x)=$



a) $-\pi\cos x$ b) $\pi\cos(\pi x)$ c) $\pi\cos\left(\frac{x}{\pi}\right)$ d) $-\pi\cos(\pi x)$ e) $-\pi\cos\left(\frac{x}{\pi}\right)$

69. If $f(x) = \tan^{-1}\left(e^{2x}\right)$ then $f'(x) =$

a) $\frac{e^x}{\sec^2 e^x}$ b) $\frac{2e^{2x}}{1+x^2}$ c) $\frac{2e^{2x}}{1+e^{4x}}$ d) $\frac{4e^{4x}}{1+e^{2x}}$ e) $\frac{e^{4x}}{1+e^{4x}}$

70. $\sin\left(2\cos^{-1}\frac{1}{4}\right) =$

a) $\frac{5}{8}$ b) $\frac{\sqrt{15}}{4}$ c) $\frac{\sqrt{15}}{8}$ d) $\frac{\sqrt{15}}{16}$ e) $\frac{7}{8}$

71. If $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$ then $f'(\sqrt{3}) =$

a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{\sqrt{3}}{4}$ e) $-\frac{\sqrt{3}}{6}$

72. The slope of the normal line to $f(x) = \tan^{-1} x$ at $x = 1$ is:

a) $\frac{\pi}{4}$ b) $-\frac{4}{\pi}$ c) $\frac{1}{2}$ d) -2 e) $-\frac{1}{\sqrt{2}}$

73. If $f(x) = \cos^{-1} 5x$ then $f'(x) =$

a) $\frac{-1}{\sqrt{1-25x^2}}$ b) $\frac{1}{\sqrt{1-25x^2}}$ c) $\frac{5}{\sqrt{1-25x^2}}$ d) $\frac{-5}{\sqrt{1-25x^2}}$ e) $-5\sin^{-1} 5x$

3rd version

① I Yes II NO (L)≠(R) III Yes (D)

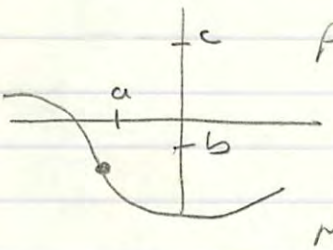
② $f(x) = \frac{a}{(x-a)^2}$ vert asym at $x=a$ elimates B, d
$f > 0 \; \forall \; a > 0$ ∴ elim c.
$\lim_{x \to \infty} f(x) \to 0$ ∴ elim E.
∴ (A) By default.

③ $\lim_{x \to -\infty} f(|x|) = +1$ (C)

④ $\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to \infty} f(x) = 3$ B C (D) E
↳ A or (D)

⑤ I $\lim_{x \to a} P(a)$ DNE True (L)≠(R)

A II P(a) NOT def, false. P(a) = b
III P(a) ≠ c while this is true, it is not the reason f is not continuous.
If P is:



P(a) ≠ c but P is cont, so not a necessary condition

⑥ I $y = 2x^{\frac{4}{3}}$ II $y = \sqrt[5]{3x^2 - 1}$ III $y = \frac{2x+5}{2x^2+5}$
Cont ∀ x ∈ ℝ, Yes all of them NO div by zero no other Domain violations.
(E)

(7) $f(x) = \begin{cases} 3x-1 & \text{if } x < -3 \\ -x^2-1 & \text{if } -3 \le x \le 2 \\ 3x+1 & \text{if } x > 2 \end{cases}$

I. cont at $x=-3$?

$f(-3) = -10$

$\lim_{x\to -3^-} f(x) = -10$

$\lim_{x\to -3^+} f(x) = -10$

Yes cont at $x=-3$

II. $f(-3) = \lim_{x\to -3} f(x)$

Yes, part of I, cont at $x=-3$

III. $f(2) = \lim_{x\to 2} f(x)$

$f(2) = -4-1 = -5$

$\lim_{x\to 2^+} f(x) = 6+1 = 7 \neq -5$

NOT TRUE.

I & II only. (D)

(8) $f(x) = x^2-4$ $g(x) = \sqrt{x}$ $f(g(x)) = |x|-4$

D_{inside} $x \ge 0$ inside set no rest $\Rightarrow$ no other rest.

(C) includes $x=0$,

Compositions cannot be continuous for values outside of its Domain.

(9) a) $f(x) = \frac{x^2+4}{(x-1)(x+4)}$ b) $f(x) = \frac{3x+1}{\sqrt{x}}$ c) $f(x) = \frac{2x-1}{x^2+1}$

D_f $x \neq \{1, -2\}$ NO

D_f $x > 0$ NO

D_f $x \in \mathbb{R}$ NO

(D) $f(x) = \frac{2x}{x^2-1}$ E) $f(x) = \frac{x^2-1}{x^2+1}$

D_f $x \neq \{\pm 1\}$ Yes

D_f $x \in \mathbb{R}$,

(10) $f(x) = \begin{cases} 2x-3 & \text{if } x > -1 \\ kx^3 & \text{if } x \le -1 \end{cases}$

$\lim_{x\to -1^-} 2x-3 \overset{\text{Set}}{=} \lim_{x\to -1^+} kx^3$

$-5 = -k$

$\therefore k = 5$ (D)

(11) $f(x) = \frac{1}{x}$ $g(x) = x^2$ $a \neq 0$

a) $\lim_{x\to a} f(g(x)) \overset{?}{=} \frac{1}{a^2}$

$\lim_{x\to a} \frac{1}{x^2} = \frac{1}{a^2}$

True

b) $\lim_{x\to a} [f \cdot g] \overset{?}{=} a$

$\lim_{x\to a} \frac{1}{x} \cdot x^2 = a$

TRUE

c) $\lim_{x\to a} f(f(x)) \overset{?}{=} \frac{1}{a}$

$\lim_{x\to a} x = \frac{1}{a}$

NO

d) $\lim_{x\to a} g(g(x)) = a^4$

$\lim_{x\to a} x^4 = a^4$

True.

e) $\lim_{x\to a} \frac{g(x)}{f(x)} \overset{?}{=} a^3$

$\lim_{x\to a} \frac{x^2}{\frac{1}{x}} = a^3$

True. (C)

(12) $y = x^3 + 3x$ $6x - y = 5$

$y = 6x - 5$

$y' = 3x^2 + 3 \overset{\text{Set}}{=} 6$

$3x^2 = 3$

$x = \pm 1$

$y = -1 - 3 = -4$ $(-1, -4)$

$y = 1 + 3 = 4$ $(1, 4)$

NOTE DO NOT use $y = 6x - 5$ to get y values. We wanted parallel Not on that line!

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

(13) $D_x(\sqrt{x+1})$ $f(x+\Delta x) = \sqrt{x+\Delta x+1}$

$D_x = \lim_{\Delta x\to 0} \frac{\sqrt{x+\Delta x+1} - \sqrt{x+1}}{\Delta x}$ (C)

(14) $y = \sqrt{x+2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x+2}}\Big|_{x=2} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$ $\perp$ $\therefore m = -4$

$y|_{x=2} = \sqrt{2+2} = \sqrt{4} = 2$ $(2,2)$ $m=-4$

$y-2 = -4(x-2)$

$y-2 = -4x+8$

$y+4x = 10$ (E).

(15) I True

II True $f'(a^+) = -\infty$ $f'(a^+) = +\infty$

III True Point.

(E) all 3.

(16) I $f(a)$ Exists True

II f cont at a NO

III f der at a NO

I only. (A)

(17) $f(x) = \underset{u}{(x^2+4)}\underset{v}{(3x^3-2)} = 3x^5+12x^3-2x^2-8$

$15x^4+36x^2-4x$

$f' = (3x^3-2)2x + (x^2+4)9x^2$

$6x^4-4x+9x^4+36x^2$

(B) $15x^4+36x-4x$

(18) $f(x) = x^{\frac{5}{3}} - x^{\frac{2}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{2}{3}x^{-\frac{1}{3}}$

$\frac{5x^{\frac{2}{3}}}{3} - \frac{2}{3x^{\frac{1}{3}}}$

$\frac{5x - 2}{3x^{\frac{1}{3}}} \overset{\text{set}}{=} 0$

$x = \frac{2}{5}$ (E)

$y = -4x - 6$

$y = 4 - 6 = -2$

$1, -2$

$-2 = 1 - 8$

$-2 = -6$

$(1, -7)$

(19) $y = x^4 - 8x$ $\quad y = -4x + 2$

$4x^3 - 8 \overset{\text{set}}{=} -4$

$4x^3 = 4$

$x^3 = 1$

$x = 1$

$y| = 1 - 8 = -7$
$x=1$

$(1, -7)$ $m = +4$ // slope $\therefore m = -4$

$y + 7 = -4(x - 1)$

$y + 7 = -4x + 4$

$y = -4x - 3$ E

(20) $f(x) = \frac{2}{x} - \frac{3}{x^2} + \frac{5}{3x^3}$

$f'(x) = -\frac{2}{x^2} + \frac{6}{x^3} - \frac{5}{x^4}$ (B)

(21) $y = \frac{1-3x}{2x-5}$ $\quad y' = \frac{(2x-5)(-3) - (1-3x)2}{(2x-5)^2}$

$= \frac{-6x + 15 - 2 + 6x}{(2x-5)^2}$

$\frac{13}{(2x-5)^2}$ (C)

(22) $\dfrac{x^2-5x+6}{x-3}$

$\dfrac{(x-2)(x-3)}{x-3} = (x-2)' = 1$, (E)

(23) $f(x) = \dfrac{x^2+2x}{x-1}$, $f' = \dfrac{(x-1)(2x+2)-(x^2+2x)}{(x-1)^2}$

$\dfrac{2x^2-2-x^2-2x}{(x-1)^2}$ (D)

$\dfrac{x^2-2x-2}{(x-1)^2}\Big|_{x=0} = -2$

(24) $f(x) = \dfrac{x^2}{3} - \dfrac{3}{x^2}$

$\dfrac{2}{3}x + \dfrac{6}{x^3}$

$\dfrac{2x^4+18}{3x^3}$ (B)

(25) $f(x) = \sqrt{x-\sqrt{x}}$ $\quad f'(x) = \frac{1}{2}(x-\sqrt{x})^{-\frac{1}{2}}\left(1-\frac{1}{2\sqrt{x}}\right)$

(C) $\quad \dfrac{1}{2\sqrt{x-\sqrt{x}}} \cdot \dfrac{2\sqrt{x}-1}{2\sqrt{x}}$

(36) $f(x) = \dfrac{x^2}{2} + x - 1$ $\quad f'(x) = x+1 \overset{\text{SET}}{=} 0$ $\quad x = -1$

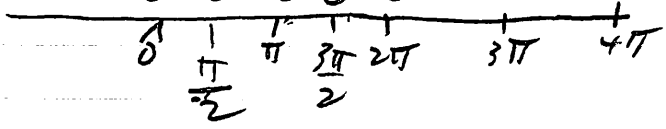
$f(-1) = \frac{1}{2} - 1 - 1 = -\frac{3}{2}$ $\therefore$ $Y = -\frac{3}{2}$ (C)

(27) $S = -3\cos 2t$

$S' = +6\sin 2t$

$2t \overset{\text{SET}}{=} k\pi$

$t = \dfrac{k\pi}{2}$ (E)

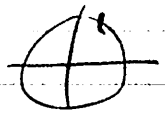


(28) $f(x) = 6\sin 2x \cos^2 x$

$= 3\sin 4x$

$f'(x) = 12\cos 4x \Big|_{x=\frac{\pi}{2}} = 12\cos 2\pi = 12.$ (D)

(29) $f(x) = 2\cos 4x$



$f'(x) = -8\sin 4x \Big|_{x=\frac{\pi}{12}} = -8\sin\frac{\pi}{3}$

$-8\,\frac{\sqrt{3}}{2} = -4\sqrt{3}$ (A)

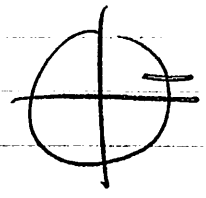
Sec "Normal" is. $\frac{1}{4\sqrt{3}} = \frac{\sqrt{3}}{12}$ (B)

(30) $f(x) = \frac{1+\sin x}{\sin x - 1}$ 4 5 $f' = \frac{(\sin x - 1)\cos x - (1+\sin x)\cos x}{(\sin x - 1)^2}$

$\frac{\sin x\cos x - \cos x - \cos x - \sin x\cos x}{(\sin x - 1)^2}$

$\frac{-2\cos x}{(\sin x - 1)^2}$ (D)

(31) $f(x) = \tan x + \sec x$



$f'(x) = \sec^2 x + \sec x \tan x \Big|_{x=\frac{\pi}{6}}$

$\frac{1}{\cos^2\frac{\pi}{6}} + \frac{1}{\cos\frac{\pi}{6}}\cdot\tan\frac{\pi}{6}$

$\left(\frac{2}{\sqrt{3}}\right)^2 + \frac{2}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}$

$\frac{4+2}{3} = 2,$ (A).

(32) $s(t) = \sin t \cos t$

$s' = (\cos t)\cos t - \sin^2 t$

$\cos^2 t - \sin^2 t$

$\cos 2t = 0$

$2t = \frac{\pi}{2}$

$t = \frac{\pi}{4}$ (C)

(33) $f(x) = (x^2 - 4x + 4)^{35}$

$(x-2)^{70}$ $\quad f'(x) = 70(x-2)^{69}$ (D).

(34) $f(x) = (3x^2 - 2x + 1)$

$f'(x) = \quad (6x-2)\Big|_{x=0} = \quad (-2) = -2$

$(0,1)$ $\quad y - 1 = -2(x)$

$y = -2x + 1$ (A)

(35) $s(t) = (3t^2 - 2t)^5$

$s' = 5(3t^2-2t)^4(6t-2)\Big|_{t=1} = 5(3-2)^4(6-2)$

$5(1)(4) = 20$ (C)

(36) $(\tan^3 2x)' = (3\tan^2 2x \sec^2 2x)2$

$6\tan^2 2x \sec^2 2x$ (D)

(37) $(x^2+1)^{-3}$

$-3(x^2+1)^{-4}(2x) = \frac{-2x}{3(x^2+1)^4}$

(B)

(38) $\lim_{h\to 0} \frac{\tan^2\left(\frac{\pi}{4}+h\right) - \tan^2\frac{\pi}{4}}{h}$ $\quad f(x) = \tan^2 x$

$f'(x) = 2\tan x \sec^2 x \Big|_{x=\frac{\pi}{4}}$

$2(1)\cdot 2 = 4$ (C)

(39) $y = \sqrt[3]{(x^2-6x+9)^2}$

$y = (x-3)^{\frac{4}{3}} \qquad y' = \frac{4}{3}(x-3)^{\frac{1}{3}}$ (B)

(40) I $f(x) = 3x^{\frac{2}{3}}$ $\quad f' = 2x^{-\frac{1}{3}}$ IS NOT

II $f(x) = 5x^{\frac{6}{5}}$ IS

III $f(x) = (x^2-1)^{\frac{3}{2}}$ Not cont at $x=0$ ∴ No Der. !! Not Defined at $x=0$!!

I, III only (D)

(41) $\left(2\sqrt{x} - \frac{3}{\sqrt{x}} + \frac{3}{\sqrt[3]{x}}\right)' = 2x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} + 3x^{-\frac{1}{3}}$

$x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}} - x^{-\frac{4}{3}}\Big|_{x=1} = 1 + \frac{3}{2} - 1 = \frac{3}{2}$ (C)

(42) $f(x) = \sin\sqrt{2x} \qquad f'(x) = \cos\sqrt{2x}\cdot\frac{1}{2\sqrt{2x}}\cdot 2$

$\frac{\cos\sqrt{2x}}{\sqrt{2x}}$ (B).

(43) $\sqrt{\frac{t^2+1}{2t-3}} = \left(\frac{t^2+1}{2t-3}\right)^{\frac{1}{2}}$ u

$\frac{1}{2}\left(\frac{2t-3}{t^2+1}\right)^{\frac{1}{2}}\left(\frac{(2t-3)2t-(t^2+1)2}{(2t-3)^2}\right)$

$\frac{1}{2}\left(\frac{2t-3}{t^2+1}\right)^{\frac{1}{2}}\left(\frac{4t^2-6t-2t^2-2}{(2t-3)^2}\right)$

$\frac{1}{2}\left(\frac{2t-3}{t^2+1}\right)^{\frac{1}{2}}\left(\frac{2t^2-6t-2}{(2t-3)^2}\right)$

$\left(\frac{2t-3}{t^2+1}\right)^{\frac{1}{2}}\left(\frac{t^2-3t-1}{(2t-3)^2}\right)$ (A)

(44) $f(1)=4 \quad f'(1)=3 \quad g(x)=\sqrt{f(x)}$

$g'(x) = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)\Big|_{x=1}$

$\frac{1}{2\sqrt{4}} \cdot 3 = \frac{3}{4}$ (B).

(45) $y = \cos^2 x - \sin^2 x = \cos 2x$

$y' = -2\sin 2x$

$y'' = -4\cos 2x$ (E).

3rd version

46. ① Inverse has segment $(1,1) \to (2,2)$ lies on $y=x$
② Set $(1,0) \to (0,1)$ because $(0,-1) \to (1,0)$
∴ (A)

47. $f(x) = \frac{x-2}{x+k}$

$x = \frac{z-2}{z+k}$

$xz + kx = z - 2$

$(x-1)z = -kx - 2$

$z = \frac{-kx-2}{x-1}$ ⇒ $\frac{-kx-2}{x-1}$ set $\frac{x-2}{x+k}$

$-kx^2 - 2x - k^2x - 2k = x^2 - 3x + 2$

$-kx^2 + (k^2 - 2)x + 2k = x^2 - 3x + 2$

$-k = 1$; $-k^2 - 2 = -3$; $-2k = 2$

$k = -1$

(C)

48.
I $f(x) = \frac{1}{x}$ Yes
II $f(x) = \frac{1}{x^3}$ NO
IV $F(y) = x$ Yes

I, IV only (D)

49. $f(x) = \frac{x+1}{x-2}$ $\quad x = \frac{z+1}{z-2}$

$xz - 2x = z + 1$

$z(x-1) = 2x + 1$

$z = \frac{2x+1}{x-1} = f^{-1}(x)$ (B)

(50) If $f(f^{-1}(x)) = x$

I $f(x) = x^3 - x^2$

$= x^2(x-1)$

$f(0) = 0$
$f(1) = 0$ ∴ NOT 1:1

II $f(x) = x^3 - 2x$

$x(x^2 - 2)$

$f(0) = 0$
$f(\sqrt{2}) = 0$ ∴ NOT 1:1

III $f(x) = x^3 + 2$

$f'(x) = 3x^2$ thus f is constantly increasing ∴ cannot achieve an earlier range value ∴ 1:1 f(x) ✓

(C)

(51) $f(x) = \frac{1}{x}$ $f(g(x)) = x$ Thus f is inverse of f. The inverse f(x) of f is $y = \frac{1}{x}$ ∴ $g = \frac{1}{x}$. (C)

(52) $f(x_0) = 2$ $f'(x_0) = 3$ f is increasing

$(f^{-1})'\Big|_{x=5} = ?$ $f'(x) = \frac{1}{g'(f(x))}$ f and g are inverses.

f($g'(5) = ?$ $g'(5) = \frac{1}{f'(g(5))}$ E

$g(5) \to ?$: $f(?) = 5$ so $? = x_0$

$= \frac{1}{f'(x_0)}$

$= \frac{1}{3}$

(53) $f(x) = \sqrt[3]{x} \Rightarrow f^{-1}(x) = x^3$

$(f^{-1}(x))' = 3x^2$ (B)

(54) $\log_{e^2} e = \frac{\ln e}{\ln e^2} = \frac{1}{2}$ (B)

(55) $f'(x) = \frac{1}{g'(f(x))}$

$g'(f(x)) = \frac{1}{f'(x)}$

we want $g'(2)$ so → what x value makes $f(x) = 2$

from table $f(1)$ is 2 so $x = 1$

want $g'(2) =$

$g'(f(1)) = \frac{1}{f'(1)} = \frac{1}{4}$ (B)

(4) ← from Table.

(56) $\ln x^4 = 4 \ln x$ True $\forall x > 0$ (C)

(57) $f(x) = \sqrt{\ln x^3} = (3 \ln x)^{\frac{1}{2}} = 3^{\frac{1}{2}} (\ln x)^{\frac{1}{2}}$

$f' = \frac{}{2\sqrt{\ln x^3}} \cdot \frac{3}{x} \quad = \text{or} = \quad 3^{\frac{1}{2}} \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$

$\frac{3}{2x\sqrt{\ln x^3}}$ $\qquad$ $\frac{\sqrt{3}}{2x\sqrt{\ln x}}$

$\frac{3}{2x\sqrt{3}\ln x} = \frac{3}{2x\sqrt{3}\sqrt{\ln x}}$

$= \frac{\sqrt{3}}{2x\sqrt{\ln x}}$ (C)

(58) $\ln|x|$ makes it all >0. (E)

(59) $f(x) = \ln(\cos 2x)$

$f'(x) = \frac{-2\sin 2x}{\cos 2x} = -2\tan 2x$ (C)

(60) $\frac{d}{dx}(\ln 3-2x)$ $\quad \frac{1}{3-2x} \cdot -2 = \frac{-2}{3-2x}$ (C)

(61) $y = \ln x| \underset{x=e^2}{} = 2 \quad (e^2, 2)$

$m = \frac{1}{x}\Big|_{x=e^2} = \frac{1}{e^2} = m$ $\quad y-2 = \frac{1}{e^2}(x-e^2)$

$y-2 = \frac{1}{e^2}x - 1$

$y = \frac{1}{e^2}x + 1$ (A)

(62) $f(x) = \overset{a}{x}\overset{b}{\ln x}$

$f'(x) = \ln x + 1 \overset{\text{set}}{=} 0$

$\ln x = -1$

$x = e^{-1}$ (E)

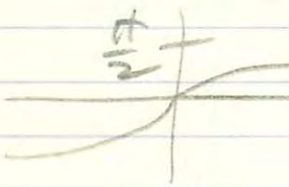
(63) $\lim_{x\to 0} \frac{\ln(2*x) - \ln 2}{x}$ $\quad$ Def of $\frac{d}{dx}\ln|_{x=2}$ $y = \ln x$

$\frac{1}{x}\Big|_{x=2} = \frac{1}{2}$. (C)

(64) $\frac{d}{dx}(\sin e^{2x})$

$\cos e^{2x} \cdot 2e^{2x} = 2e^{2x}\cos e^{2x}$ (B)

(65) $\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$ (D)



(66) $\tan(\sin^{-1}(\frac{2}{3}) + \csc^{-1} 3) = \tan(\sin^{-1}(\frac{2}{3}) + \sin^{-1}(\frac{1}{3}))$

$\frac{\tan(\sin^{-1}\frac{2}{3}) + \tan(\sin^{-1}\frac{1}{3})}{1 - \tan(\sin^{-1}\frac{2}{3})\tan(\sin^{-1}\frac{1}{3})}$

$\tan(\sin^{-1}x) = \frac{\sin(\sin^{-1}x)}{\cos(\sin^{-1}x)} = \frac{x}{\sqrt{1-x^2}}$

$\tan\sin^{-1}(\frac{2}{3}) = \frac{\frac{2}{3}}{\sqrt{1-\frac{4}{9}}} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{\sqrt{5}}$

$\tan\sin^{-1}(\frac{1}{3}) = \frac{\frac{1}{3}}{\sqrt{1-\frac{1}{9}}} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{2\sqrt{2}}$

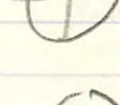
$\frac{\frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{2}}}{1 - \frac{2}{\sqrt{5}}\cdot\frac{1}{2\sqrt{2}}} \quad \frac{\frac{4\sqrt{2}+\sqrt{5}}{2\sqrt{10}}}{\frac{2\sqrt{10}-2}{2\sqrt{10}}} = \frac{4\sqrt{2}+\sqrt{5}}{2\sqrt{10}-2}$

(D)

(67) $\tan\theta = 2$

$\tan(\frac{\pi}{3}+\theta) =$

$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$



$\frac{\sqrt{3}+2}{1-2\sqrt{3}}$ (C).

(68) Amp $= \pi$, Negative.

Per $= 2 = \frac{2\pi}{b}$ $\quad b = \pi$

$f(x) = -\pi \cos \pi x$ (D)

(69) $f(x) = \tan^{-1}(e^{2x})$ $\quad f' = \frac{1}{1+e^{4x}} \cdot 2e^{2x}$

$\frac{2e^{2x}}{1+e^{4x}}$ (C)

(70) $\sin(2\cos^{-1}\frac{1}{4}) = 2\sin\cos^{-1}\frac{1}{4}\cos(\cos^{-1}\frac{1}{4})$

$2\sqrt{1-\frac{1}{16}} \cdot \frac{1}{4}$

$2\frac{\sqrt{15}}{4} \cdot \frac{1}{4} = \frac{2\sqrt{15}}{16} = \frac{\sqrt{15}}{8}$ (C)

(71) $f(x) = \tan^{-1}(\frac{1}{x})$ $\quad f'(x) = \frac{1}{1+\frac{1}{x^2}} \cdot \frac{-1}{x^2}$

$\frac{-1}{\frac{x^2+1}{x^2}} \cdot \frac{1}{x^2} = -\frac{1}{x^2+1}\Big| = -\frac{1}{4}$ (B)

$x=\sqrt{3}$

(72) $f(x) = \tan^{-1}x$ $\quad f'(x) = \frac{1}{1+x^2}\Big| = \frac{1}{2}$

$\perp \therefore m = -2$ (D) $\qquad x=1$

(73) $f(x) = \cos^{-1}5x$ $\quad \frac{d}{dx}\cos^{-1}A = \frac{-1}{\sqrt{1-A^2}} \cdot dA$

$\frac{-5}{\sqrt{1-25x^2}}$ (D)